COSMIC ACCELERATION FROM CAUSAL BACKREACTION IN A SMOOTHLY INHOMOGENEOUS UNIVERSE

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ABSTRACT

A phenomenological formalism is presented in which the apparent acceleration of the universe is generated by large-scale structure formation, thus eliminating the coincidence and magnitude fine-tuning problems of the Cosmological Constant in the Concordance Model, as well as potential instability issues with dynamical Dark Energy. The observed acceleration results from the combined effect of innumerable local perturbations, due to individually virialized systems, overlapping together in a smoothly-inhomogeneous adjustment of the FRW metric, in a process governed by the causal flow of inhomogeneity information outward from each clumped system. We discuss several arguments from the literature claiming to place sharp limits upon the strength of backreaction-related effects, and show why such arguments are not applicable in a physically realistic cosmological analysis. A selection of simply-parameterized models are presented, including several which are capable of fitting the luminosity distance data from Type Ia supernovae essentially as well as the best-fit flat ΛCDM model, without resort to Dark Energy, any modification to gravity, or a local void. Simultaneously, these models can reproduce measured cosmological parameters such as the age of the universe, the matter density required for spatial flatness, the present-day deceleration parameter, and the angular scale of the Cosmic Microwave Background to within a reasonable proximity of their Concordance values. We conclude by considering potential observational signatures for distinguishing this cosmological formalism from ΛCDM or Dark Energy, as well as the possible long-term fate of such a universe with ever-spreading spheres of influence for its increasingly superposed perturbations.

Subject headings: cosmological parameters — cosmology: theory — dark energy — large-scale structure of universe

1. INTRODUCTION AND MOTIVATIONS

During the past decade or so, a number of complementary observational techniques have come together to point towards the existence of an exotic cosmic ingredient known as Dark Energy (DE). Luminosity distance measurements from Type Ia supernovae (Perlmutter et al. 1999; Riess et al. 1998) bear evidence of an apparent cosmic acceleration, which seems to indicate the existence of a substance capable of violating the strong energy condition (Hawking & Ellis 1973), and perhaps even the dominant energy condition (Caldwell 2002; Caldwell et al. 2003). Observations of the Cosmic Microwave Background (CMB) indicate a spatially flat universe (Komatsu et al. 2009); yet various lines of evidence regarding large-scale structure formation and other measurements (e.g., Turner 2002a) indicate a total dynamical matter content of only $\Omega_M \sim 0.3$, far less than the value of unity needed for flatness. The standard conclusion from this, barring modifications to gravity, is that there must exist a smooth (non-clumping) component to the inventory of the universe supplying the remaining $\Omega_{DE} \sim 0.7$, which is capable of accelerating the universe without joining in the process of localized structure formation.

Within the current resolution of the data, combinations of observations are still consistent (e.g., Komatsu et al. 2009) with the simplest Dark Energy model: flat ΛCDM with a Cosmological Constant (vacuum energy) as the Dark Energy 'substance'. A nonzero Λ density would clearly avoid participating in spatial clustering, while also possessing an equation of state, $w \equiv w_0 = -1$, that would begin to drive the acceleration once Ω_Λ had become the dominant cosmic component.

There are well-known aesthetic problems with Λ as the missing piece to the puzzle, however. This includes a fine-tuning problem in which one must explain why ρ_Λ is some ~ 120 orders of magnitude smaller than what would naively be expected from the Planck scale (e.g., Kolb & Turner 1990). And perhaps even more relevant to issues of cosmic evolution, there is the "Coincidence Problem" (e.g., Arkani-Hamed et al. 2000), which questions why, given that $\rho_\Lambda/\rho_M \propto a^3$, that observers *today* should just happen to live right around the time (within a factor of ~ 2 in scale factor) when Λ began dominating the cosmic evolution. After all, if ρ_Λ had been just a little smaller, then cosmologists in this epoch would have had no hope of detecting it; and if ρ_Λ had been just a little larger, then the acceleration would have prevented structure formation from ever having occurred, and no observers would exist to measure it at all.

Without delving deeply into anthropic issues, one possible way of avoiding such problems is to give the Dark Energy an evolution in $w(z)$, where such coincidences are eliminated or moderated; an example being tracker quintessence solutions (Zlatev et al. 1999; Albrecht & Skordis 2000), where there are natural reasons for Dark Energy to 'pop up' at

a propitious, recent (relative to the Planck scale) time – such as being triggered by a phase transition like the onset of matter domination, for example. Moving away from a Cosmological Constant by giving the Dark Energy an evolving equation of state, however, also opens the door to other possible dynamics – such as the ability for the Dark Energy to be mobile and become spatially inhomogeneous (Caldwell et al. 1998). A dynamical Dark Energy (DDE) of this type is liable to join in structure formation to an observationally unacceptable degree, thus ruining it as a candidate to serve as the smoothly-distributed missing cosmic ingredient.

An under-appreciated aspect of the notion of Dark Energy is what it really means when something is a 'negative pressure' substance. Thinking of the acceleration of the cosmic expansion, it is common to refer to the DE with words like "repulsive", "antigravity", and so on (e.g., Heavens 2010), as if negative pressure substances were materials that *naturally* did not possess attractive forces or tend to clump on local scales, thus making them natural candidates for being a smooth cosmic component. But this is the exact opposite of the truth: a simple look at the 1st Law of Thermodynamics, $P = -\partial[\text{Energy}]/\partial[\text{Volume}]$, shows that for a negative pressure ($P < 0$) substance, an increase in system volume requires an increase in its energy content ($\partial E/\partial V > 0$). Consequently, since the environment must do work upon a local patch of DE to make it expand (and gets work back when letting it contract), a material with negative pressure should be *self-attractive*, not self-repulsive. Thus a true negative pressure species which is mobile (i.e., not Λ), able to move around and clump due to its own self-attractive forces, would naturally be expected to not only *fail* to remain smooth on galaxy cluster scales, but due to its extreme ($w \equiv P/\rho \sim -1$) negative pressure, it would clump *relativistically*, making it the most strongly clumping material in the universe!

Being self-attractive would seem to be a counterintuitive property to have for a substance that is invoked in order to make the universe increase its expansion rate and thereby spread out faster, but in fact it is due to a sign peculiarity in the FLRW acceleration equation (e.g., Kolb & Turner 1990):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \; , \tag{1}$$

where $a(t)$ is the cosmic scale factor. From the negative sign on the right-hand side of this equation, we see that negative pressure does in fact cause a cosmic acceleration, not a deceleration; and thus we have the seemingly paradoxical (but true) result that a force which attracts *locally*, repels *globally*. This qualitative view is supported by the work of Maor & Lahav (2005) and Maor (2007), where they found that for a two-component system made up of matter and dark energy, the virialization radius will increase when the Dark Energy is allowed to dynamically participate in the virialization, a sign of an additional attractive force (due to the increased virial kinetic energies occurring in a deeper poten-

tial well). One implication of this is that signs of a cosmologically recent slow-down in the growth of clustering (e.g., Vikhlinin et al. 2009), findings which are viewed as strong evidence in favor of Dark Energy, really only support a Cosmological Constant form of DE, not any more general DDE material with locally-attractive negative pressure. It therefore seems that a negative pressure substance like dynamical Dark Energy would be exactly the wrong prescription when looking for a cosmic component that can remain smoothly distributed in order to avoid enhancing the small-scale clustering of material – thus indicating an essential conflict between the dual requirements of smoothness and acceleration for just about anything other than the immobile, coincidence-prone Λ.

This potential instability of DDE to spatial perturbations is well known by experts in the field (e.g., Turner & White 1997; Hu 1998, 2005); and various forms of support pressure may be proposed in order to subvert the natural behavior of negative pressure substances towards small-scale clumping, thus allowing a DDE to remain smooth on scales large enough to evade limits on Ω_M from observations of large-scale structure formation. But beating the laws of thermodynamics is a tricky business to be in, and often one errs by implicitly introducing a source of *positive* pressure somewhere in the problem – perhaps within the Dark Energy itself, such as by considering relativistic particles (with their ordinary, thermodynamic positive pressure) as the DDE (Turner & White 1997); or by using the (positive) degeneracy pressure of long-wavelength particles like neutrinos to keep them unclumped (e.g., Wigmans 2004). Or alternatively, positive pressure can be generated by an auxiliary material invoked in combination with the DDE to keep the latter from clustering (e.g., Bjælde et al. 2008). Either way, a positive pressure source sufficient for supporting the DDE against collapse must invariably contribute a *decelerating* contribution to the cosmic expansion (cf. Equation 1) that counteracts the accelerative properties for which the Dark Energy was recruited in the first place.

Even without any implicit, possibly inadvertent introduction of a positive (adiabatic) pressure source, it could still be argued that some form of *nonadiabatic* pressure within the DDE might succeed at preventing its small-scale clustering without ruining the actual cosmic acceleration. For a scalar field, it is well known (Grishchuk 1994; Caldwell et al. 1998) that the sound speed is scale-dependent; and despite being imaginary (thus unstable against collapse) with $c_s^2 < 0$ for long-wavelength perturbations of a $w < 0$ scalar field, for small wavelengths the sound speed instead approaches $c_s^2 \simeq c^2$, thus seemingly solving the instability problem automatically when considering small-scale clustering. But the situation is not quite so simple, however, when considering the detailed processes of inhomogeneous structure formation. For example, in a study modeling the spatial evolution of scalar field DDE in the presence of a spherically-collapsing matter overdensity, Dutta & Maor (2007) find that the initial response of the DDE is to join the matter in entering into a collapsing

phase, thus adding to the growing central overdensity. This collapsing phase does not last long, since the slower Hubble expansion in the central matter overdensity allows the scalar field to locally roll down its potential and lose energy, actually creating a weak ($\delta\rho_\phi/\rho_\phi \sim -0.02$) *void* of DDE there, thus appearing once again to solve the small-scale DDE clustering problem. But recalling the 1$^{\text{st}}$ Law of Thermodynamics ($P = -\partial E/\partial V$), a material that loses energy when the universe expands is more of a positive pressure substance than a negative pressure substance; and sure enough, Dutta & Maor (2007) find that the effective equation of state w of their DDE does become substantially less negative as the system evolves, particularly in the center where the DDE void is developing. Thus we see that the less a DDE engages in small-scale clustering, the less it behaves like a negative-pressure substance capable of accelerating the universe; and it is not unreasonable to suppose that these two properties are fundamentally connected in a thermodynamically essential way. Now, we should not overstate the case that can be made from those results, since the scalar fields examined in Dutta & Maor (2007) never actually lost their accelerative nature (i.e., $w \geq -1/3$ is never reached): the equation of state only increases from $w \equiv -1$ (everywhere) at $z = 35$ to $w \simeq -0.8$ (at the center) at $z = 0$ for their simplest scalar field potential (and increases only to $w \simeq -0.945$ for their more theoretically-motivated double exponential potential). But then again, their numerical analysis only studied the system within the regime of linear perturbations; and it would be interesting to see what happens to the averaged equation of state of the total cosmic contents (matter plus DDE) – in order to evaluate the actual ability of such a cosmic mixture to effectively accelerate the universe – when a significant fraction of the DDE is located within regions of highly nonlinearly collapsing matter overdensities. (While Mota et al. (2008) do perform a analysis that is fully nonlinear for the matter perturbations, they do not evaluate any effective changes to w which would occur as a result. Furthermore, the assumption made in both of these papers of a very small mass scale for the DDE scalar field, $m_\phi \sim O(H_0)$, may inadvertently be recruiting degeneracy or relativistic pressure within the DDE itself to help keep it unclumped on small scales; this might explain the seemingly odd result in Mota et al. (2008) that the DDE cannot clump onto pre-existing overdensities by itself, but instead stops increasing its density contrast once the nonlinear matter overdensity pulling the DDE in stabilizes its own collapse through virialization of the matter alone.)

And even beyond the question of changes to the effective equation of state, w, there is a more subtle issue involved here as well, which we pose as the following question: if the *clustering* properties of a substance with nonadiabatic pressure do not solely depend upon w, then why would its *accelerative* properties depend solely upon w? In the textbook derivation (e.g., Kolb & Turner 1990) of the acceleration equation, Equation 1, one proceeds by first assuming a perfect-fluid form for all cosmic constituents; but a DDE with its inhomogeneities

controlled by nonadiabatic pressure would clearly fail to behave as a perfect fluid, placing the very simple form of the FLRW acceleration equation itself into question (as well as muddying the physical meaning of the equation of state parameter, $w \equiv P/\rho$). If one cannot trust the DDE to behave as a perfect fluid in one sense, then there is less reason to trust it to behave as a perfect fluid in any other sense; and thus it seems quite reasonable to believe that a dynamical substance sufficiently exotic as to avoid clumping despite $P < 0$, might *also* fail to generate $\ddot{a} > 0$ despite $P < -\rho/3$. Rather than simply quoting a sufficiently negative value of w as demonstrating a claim that some non-clustering DDE is capable of accelerating the universe, a fully general-relativistic treatment of all of the inhomogeneous-DDE-induced accelerative and/or decelerative effects within a fair cosmological sample (clusters and voids) would need to be done to properly substantiate that claim.

While the foregoing discussion does not constitute actual proof against the possibility of achieving acceleration with a negative pressure, mobile substance that somehow fails to clump, it should at least introduce a degree of skepticism into an idea that has generally been accepted at face value. And if these concerns about coincidences and negative-pressure instabilities do in fact cause one to abandon both Λ and DDE, then explaining the cosmic acceleration would seem to present a conundrum. Fortunately, however, the self-attractive nature of negative pressure presents us with the seed of an alternative idea: in its own way, *normal gravitational attraction* represents a form of negative pressure – specifically, the very same gravitational attraction which is involved in the growth of galaxies and galaxy clusters in large-scale structure formation. One might therefore make a virtue of necessity, and try to recruit structure formation itself as the driver (not just the trigger, as in some tracker quintessence models) of the cosmic acceleration. Such a step would completely solve the Coincidence Problem, since the remarkably contemporary onsets of both the acceleration and the existence of galaxies (and hence planets, life, and astronomical observers) could be viewed as nothing more coincidental than finding two neighboring apples that have fallen from the same tree.

It was the realization of the paradoxically accelerative yet locally attractive nature of $P < 0$ for Dark Energy that initially led this author to search for a structure-formation-induced solution to the cosmic acceleration problem (Bochner 2007a). During the past few years, a number of other researchers have also proposed a variety of methods (e.g., Schwarz 2002; Räsänen 2004; Kolb et al. 2005a; Wiltshire 2007, as a few examples) of trying to obtain the observed cosmic acceleration from the breaking of FLRW homogeneity and the formation of structure, without requiring the intervention of non-standard gravitational or particle physics, or the non-Copernican approach of placing us near the center of a large cosmic void. This type of effort has generally become known as "backreaction". The backreaction paradigm, in the consensus view, has so far been unable to completely replace Dark Energy

as the source of the apparent acceleration (we will discuss some of the reasons for this belief in the next section); and the arguments which this author originally presented in Bochner (2007a) were advanced without the benefit of a plausible mechanism for realistically generating an acceleration.

In this paper, however, we will introduce a physically plausible mechanism for producing an observed acceleration; and though our method here is strictly phenomenological, it serves as the basis for a formalism that we can use to not only reproduce the cosmic acceleration (as indicated by observations of standard candles such as Type Ia supernovae), but also, simultaneously, to reproduce several of the other key features of the apparent ΛCDM concordance.

This paper will be organized as follows: in Section 2, we discuss several of the oft-quoted limitations of the backreaction mechanism, countering them, and then use such arguments as guideposts towards developing our formalism in Section 3; in Section 4, we describe our specific models and present our simulated Hubble curves; in Section 5, we show how to extend these results to the formulation of a new "Cosmic Concordance"; in Section 6, we evaluate the success of this alternative concordance, and then suggest potential methods for observationally testing our paradigm in order to distinguish it from ΛCDM (and perhaps also from DDE); in Section 7, we discuss the possible long-term futures ('fates') of a universe dominated by our "causal backreaction" mechanism; and we conclude with a summary of these ideas and results in Section 8.

2. VARIETIES OF "BACKREACTION" AND THEIR COSMOLOGICAL IMPACT

There are several distinct ways of achieving an observation that looks like a cosmic acceleration, and one may categorize the different approaches or scenarios in various different ways (see, for example, Biswas et al. 2007; Biswas & Notari 2008). Here we will find it useful to characterize the various forms of observed acceleration according to the (q_1, q_2, q_3, q_4) terminology from Hirata & Seljak (2005) – where q_1 refers to an 'actual' acceleration as described by an increase in the volume expansion θ; q_2 refers to an 'averaged' acceleration for inhomogeneous spacetimes, an effect strictly due to nonlinearities in general relativity (GR) when one has departures from Newtonian metric perturbations (i.e., the "fitting problem", Ellis 1984; Ellis & Stoeger 1987); and q_3, q_4 refer to an *apparent* acceleration, due to the limitations that one always experiences in the act of observation, attempting to infer the general cosmic evolutionary behavior from a circumscribed set of luminosity distance measurements. (The difference between q_3 and q_4 has to do with how they perform angle-averaging for an

anisotropic universe – for example, regarding the possibility of finding 'acceleration' in some directions, and 'deceleration' in others.)

In general, q_3 and q_4 may be strongly dependent upon the observer's position in space, in relation to inhomogeneities, voids, etc., as well as being affected by anisotropic observations. For our formalism, however, which we will be describing as "smoothly inhomogeneous", the observed acceleration will depend neither upon the position of the observer nor the direction in which one looks, but will instead depend solely upon the *time* of the observation. Technically speaking, only q_2 – which relies directly upon the nonlinear character of GR – was originally termed "backreaction", although common usage has generally broadened the term to mean any form of observed or apparent acceleration from cosmic inhomogeneity. In this paper, we will use the term in its broadest sense, to mean any or all of (q_1, q_2, q_3, q_4). In the full complexity of GR for an inhomogeneous universe, it is not always a simple matter to disentangle these different terms in a clear-cut fashion; but our formalism presented here is likely best described as being the ultimate result of an averaged q_1-type volume creation, perhaps in combination with q_3/q_4-type effects due to the fact that gravitational perturbation information coming in from far away – and thus from earlier look-back times, when the universe was less clustered – actually allows the late-time/small-distance cosmic expansionary behavior to be appear quite different from what we see from far away, but only in a purely observational sense (i.e., not due to any real local differences), and in a way that *any* observer would see for their own local cosmic neighborhood.

At this present moment in the general theoretical consensus, backreaction is not widely considered to be a viable method of achieving the observed acceleration (e.g., Schwarz 2010). There are a number of important reasons for this, which seem on the surface to be strong arguments; but we will explain here why despite being technically correct, several of those arguments do not properly apply to a physically realistic cosmological model.

2.1. Smoothly-Inhomogeneous Backreaction: No Voids, Holes, or Special Boundary Conditions

One simple way of achieving an apparent acceleration via FRW-violating inhomogeneities, is to imagine ourselves located within an underdense void (e.g., Tomita 2001; Moffat 2005; Alnes et al. 2006). Such a void, expanding at a faster rate than the average cosmic expansion overall, would imprint Hubble curves with what looks like the signs of a recent cosmic acceleration, as incoming light signals from standard candles cross over from slower-expanding regions farther away from us, to the faster-expanding regions inside our local void. Such a void solution has difficulties, however – including a coincidence problem of

its own, regarding our fairly suspicious (if random) position very close to the center of the void, as would be needed to maximize 'acceleration' effects while minimizing the resulting anisotropies imposed upon the observed CMB radiation; as well as the necessary existence of a substantially large and deep void that would be capable of achieving sufficient apparent acceleration (Alexander et al. 2009). The existence of such a void would eventually become detectable using luminosity distance data over intermediate redshifts (Clifton et al. 2008); and though it has been argued that a variety of cosmological observations remained compatible with the possibility of such a large local void (Garcia-Bellido & Haugbølle 2008), recent Type Ia supernova data is consistent with a determination that no so-called "Hubble bubble" exists (Hicken et al. 2009b), a rejection of the void hypothesis further substantiated by new Cepheid data used for calibrating such supernovae (Riess et al. 2011).

Perhaps fortunately, therefore, our formalism does not require the existence of a special local void centered upon us, but instead uses the overall, averaged effects of *many* inhomogeneities, all summed together, in order to achieve the desired observational result. But the question necessarily arises as to how to represent a situation with multiple inhomogeneities via an appropriate cosmological model. One answer to this difficulty that has been studied in detail is a class of models known as "Swiss-Cheese" solutions, in which one starts with a homogeneous FRW background (the "cheese"), and carves out spherical regions (the "holes") in which matter can be distributed inhomogeneously – though still spherically symmetrically – in such a way as to represent many independent clumped structures of matter. One can use any spherically symmetric (dust-only) metric that one wishes for modeling the holes, such as the static (fully collapsed) Schwarzschild metric, as used in the original Einstein-Straus solution (Einstein & Straus 1945); or the radially-evolving LTB metrics (Lemaître 1933; Tolman 1934; Bondi 1947), as used in various recent studies (e.g., Biswas & Notari 2008). The key here is that by retaining spherical symmetry – thus recruiting Birkhoff's Theorem (e.g., Weinberg 1972) – and by carefully matching the mass-energy content of the hole to the exterior cheese at the hole boundary, one aims to completely zero out the effects of the existence of the hole on its surroundings. Thus the exterior regions continue to expand exactly as they would in an entirely homogeneous FRW cosmology, as if no holes existed, while giving one the ability to produce exact solutions for detailed study which do include deviations from homogeneity. Since each hole is presumed to have no effect on anything exterior to itself, one can include any number or variety of holes (as long as they do not intersect one another), even to the extent of filling the universe *entirely* with holes (and thus with essentially no cheese anywhere) in a fractal-like pattern, such as the "Apollonian gasket" (Marra et al. 2007). Such a model would have the same average expansion as a FRW-everywhere model, despite actually being FRW virtually nowhere.

Despite being mathematically convenient for study, however, such models have serious

disadvantages. One key shortcoming is that since they obey FRW dynamics on average, and since the holes have no effect on the exterior cheese, there is by definition zero actual "backreaction": all effects causing apparent deviations from matter-dominated FRW expansion are purely observational, due to effects upon the photons passing through the holes. Obviously, this will inhibit the ability of the inhomogeneities to mimic a cosmic acceleration; and Biswas & Notari (2008) find that such effects are indeed strongly suppressed, by a factor $(L/R_H)^3$ for passage through a hole, where L is the size of the hole and R_H is the Hubble radius. Even if one integrates the paths of light rays through many holes, this is clearly too small of an effect to replace the Dark Energy.

Such suppression of backreaction effects is entirely artificial, however, since the Swiss-Cheese models themselves are artificial – they are, in fact, entirely inadequate for plausible studies of the real universe, since they lack so many key physical features. First, the spherical symmetry of the holes makes them unable to incorporate the virialization and stabilization of forming structures (a point noted by Biswas & Notari (2008), and which will take on more importance below). Second, the prescription of taking all effects of the hole back upon the exterior universe and setting them to *zero* is entirely unphysical – when condensed structures form in the real universe, rather than leaving exterior regions alone, they instead tend to become overdense attractors which keep growing in mass and perturbative cosmological influence; consider for example our acceleration towards the Great Attractor/Shapley Concentration (e.g., Lucey et al. 2005; Bolejko & Hellaby 2008; Colin et al. 2010), and other evidence of large-scale bulk flows (Feldman et al. 2010). Third, the notion of disjoint, non-intersecting holes – a necessary assumption for producing exact metric solutions, requiring special and implausible boundary conditions at the hole/cheese interfaces – is a fatal failure of the model, since in the real universe, the spheres of influence of individual inhomogeneities always continue to expand as they pull in more material (and have more time to spread their causal influences), until the separate perturbations eventually *overlap and merge*. The superposition of gravitational perturbations from many different inhomogeneities is a phenomenon that Swiss-Cheese models cannot represent at all. Simply speaking, if we recall the earlier suppressive factor of $(L/R_H)^3$ – and now consider that each hole (overlapping with all of the other holes) is about the size of the observational horizon out to which its gravitational pull can be causally felt – then this factor is much closer to unity, and not much of a 'suppression' after all.

The challenge, of course, is how to represent such a model mathematically; since once the holes all merge together, it seems like we are back to the homogeneous FRW model again, in some averaged sense. The metric that one uses, in any case, cannot depend upon spatial position, since we are only considering averaged effects. But what we will justify below, in this section – and present a phenomenological model for, in Section 3 – is that the

perturbations can indeed be perpetually nontrivial functions of time (and of time alone); and the fact that we live merely in an averaged FRW universe (instead of in a truly homogeneous FRW universe) can be modeled as alterations to the behavior of the average scale factor, $a(t)$, in such a way that it looks like we live in a FLRW universe with matter and Dark Energy, rather than dust alone. This model, in which we re-establish the mathematical homogeneity and isotropy of the Cosmological Principle, while retaining the average, integrated effects of many overlapping inhomogeneities (incorporated as a perturbation to the Friedmann expansion), is what we refer to as a "smoothly-inhomogeneous" universe. As will be shown in Section 4, these effects are capable (under realistic conditions) of reproducing the entire observed acceleration; and an important benefit of the smoothly-inhomogeneous model is that there is nothing special about our own position, and so *all* observers (barring strong local influences) would see the *same* apparent acceleration at the same cosmic time.

2.2. Vorticity as a Large-Scale Player

During the evolution from 'truly homogeneous' to 'smoothly inhomogeneous', a strong phase transition takes place throughout the entire universe. The question is whether or not the effects of this phase transition would have large 'accelerative' effects – or as some have argued, whether or not it can have *any* accelerative effects. Sharp limits have been placed upon such mechanisms by arguments in the literature, and so we must explain here why those limits are not applicable to realistic situations.

It has been argued, for example (Alnes et al. 2007), that cosmologies dominated by pressureless dust do not permit acceleration (i.e., true volumetric acceleration, in the sense of q_1). This conclusion was based upon a study of LTB inhomogeneity, which as we have noted is limited to spherical symmetry, and lacks the ability to model virialization; but, what is the true importance of virialization? Consider the q_1 acceleration "no-go" theorem of Hirata & Seljak (2005), that was based upon the Raychaudhuri equation (Hawking & Ellis 1973), which assuming perfect fluids can be written as:

$$\frac{d\theta}{dt} = -\frac{\theta^2}{3} - \sigma_{\mu\nu}\sigma^{\mu\nu} - 4\pi G(\rho + 3P) + \omega_{\mu\nu}\omega^{\mu\nu} \; , \tag{2}$$

where θ is the volume expansion, $\sigma_{\mu\nu}$ is the shear tensor, $\omega_{\mu\nu}$ is the vorticity tensor, and ρ and P are, respectively, the density and the isotropic pressure of all material contained within the stress-energy tensor $T_{\mu\nu}$. From this expression, it is obvious that if the vorticity is zero (or more precisely, if $\omega^2 \equiv \omega_{\mu\nu}\omega^{\mu\nu}/2 = 0$), and if the strong energy condition (SEC) is obeyed ($\rho + 3P \geq 0$) for all species, then clearly $d\theta/dt \leq 0$, and no acceleration is possible. On the other hand, any violation of the SEC due to negative pressure such that $\rho + 3P \leq 0$

(i.e., $w \leq -1/3$), strong enough to lead to $d\theta/dt > 0$, would necessarily be classed as a form of Quintessence, Dark (or Phantom) Energy. Therefore, in a matter-dominated universe with negligible pressure ($P \approx 0$), with this dust being largely irrotational ($\omega^2 \approx 0$), all of the remaining nonzero terms would provide only negative (decelerative) contributions to ensure that $d\theta/dt \leq 0$.

Here, however, is one of the key questions of this paper: *Why is the vorticity set to zero?* This seems to be an especially unusual prescription in a universe where nearly everything is rotating and/or revolving about something. After all, all virialized structures in the universe larger than individual solid objects[1] are stabilized against collapse by *some* version of vorticity or velocity dispersion. Henceforth referring to all varieties of such mechanisms as simply "vorticity", this includes both the organized rotational motions of solar system planets and spiral galaxies, as well as the less organized orbital motions of stars in elliptical galaxies, and of individual galaxies in galaxy clusters. The virialization mechanism itself (e.g., violent relaxation; Lynden-Bell 1967; Shu 1978) achieves such stabilizations by generating or concentrating all of this vorticity. The value of ω^2 within galaxies and galaxy clusters must obviously be large – and in fact, physically dominant – since it is clearly sufficient to counter the gravitational attraction of the matter (including Dark Matter) to produce static regions of space with $d\theta/dt = 0$.

It is certainly a bad physical approximation to take the dominant physical force (in opposition to gravity) that regulates cosmic structure formation[2], and set it equal to zero. We must therefore consider the reasons for which this is typically done. Mathematically, zero vorticity is necessary for the existence of the synchronous comoving gauge (e.g., Hirata & Seljak 2005), such as is used for the homogeneous and isotropic FLRW cosmologies; and vorticity causes problems with the meshing together of the hypersurfaces of proper time (e.g., Räsänen 2006). Thus one cannot define an idealized "Hubble flow" unless one sets the rotation to vanish; and so to use this common and useful simplification, it must be assumed that on large enough scales, one is justified in averaging any cosmological fluid to an irrotational, comoving state. In essence, therefore, incorporating vorticity explicitly would cause extreme complications for the entire mathematical machinery of how cosmological evolution

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[1]Solid objects are supported by their internal pressure gradients and body forces, which also contribute (though likely on a cosmologically small level) to $d\theta/dt > 0$, but which have been dropped even before getting to Equation 2 as a result of the perfect fluid approximation.

[2]All of this vorticity requires great amounts of motion – i.e., kinetic energy. This energy comes from the gravitational potential energy of inhomogeneities originally pulled apart by the expanding universe. Thus the matter is 'pumped' by the expansion; and this accumulation of energy taken from the expanding universe is how the gravitationally-self-attractive cosmic matter effectively behaves like a 'negative pressure substance'.

and structure formation are usually studied in practice[3]. Such complications, barring the unlikely possibility of rotation on supercluster or even larger scales, would appear to be physically unnecessary to keep track of on a truly cosmological level. Thus vorticity has in many cases been relegated to lesser status as a "small-scale player" (e.g., Buchert 2008), relevant only for cosmic averages performed over domains that are on or below the scales of galaxy clusters.

The problem with this approach is that it is not actually the vorticity tensor, $\omega_{\mu\nu}$, which appears in the Raychaudhuri equation as a positive contributor to $d\theta/dt$, but the *square* of the vorticity, ω^2. And no matter what scale one goes up to in their averaging, the square of a quantity will *never* average away; in fact, the total amount of integrated ω^2 in a volume will be strictly proportional to volume, if the clustering and stabilization of mass is essentially the same everywhere in space. As Buchert & Ehlers (1997) put it, cosmic averages of positive semi-definite quantities like $\langle\omega^2\rangle$ (or in opposition to it, $\langle\sigma^2\rangle$) get "frozen" at the size of "typical subdomains"; and thus they cannot be made to go to zero by averaging over larger domains, even if the spatially averaged value of the parent tensor, $\langle\omega_{\mu\nu}\rangle$, itself becomes negligible on large scales due to collective cancellation from neighboring local subregions in the domain. Furthermore, the evolving cosmic value of $\langle\omega^2\rangle$ is not subject to any particular conservation law (since $\omega_{\mu\nu}$ throughout the early universe was not exactly zero), nor is ω^2 limited from growing as large as it needs to be in any location in order to virialize a self-stabilizing region of clustered mass – formally speaking, Asada & Kasai (1999) find that vorticity is coupled to density contrast, becoming strongly amplified in locally collapsing regions, contrary to the expectations from standard linear perturbation theory. Systems demonstrating significant vorticity-dependent behavior should be quite natural in the real universe, as depicted generically in Figure 1 (with angular momentum and its squared value, $\boldsymbol{L}$ and L^2 respectively, serving as a proxy for $\omega_{\mu\nu}$, ω^2 here). We can even observe the results of this kind of behavior in our nearby cosmic neighborhood by looking at the Local Group, in which the Milky Way and Andromeda galaxies each support themselves against collapse via their own individual spins; and yet Andromeda and the Milky Way, like many pairs of large spiral galaxies, are counter-rotating with respect to one other (Schaaf 1975, p.174), thus partially canceling $\langle\omega_{\mu\nu}\rangle$ for the Local Group as a whole (though not necessarily implying

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[3]There are of course a few analyses which do not immediately drop cosmological vorticity at the outset of calculations: e.g., Ehlers & Buchert (1997), Asada & Kasai (1999), Christopherson et al. (2009), and Räsänen (2010), as a few examples. But this does not guarantee that vorticity is being included to the degree necessary for generating causal backreaction. For example, Räsänen (2010) finds the relevant contribution due to vorticity to be simply a total divergence, leading only to a negligible surface term; but they restrict their analysis to first order in velocity v – and as will be made clear below in Section 2.3, an analysis at least up to $O(v^2)$ is necessary in order to incorporate the causal flow of gravitational information.

that it is small or conserved per se – e.g., Dunn & Laflamme (1993)), without reducing its total integrated value of $\langle \omega^2 \rangle$.

Given the potential significance of vorticity in regards to both astrophysics and volume expansion, general warnings against neglecting it are given (multiple times in fact) by Räsänen (2006) (and by various other authors), in the context of acceleration via structure formation; but aside from such caveats, for typical studies in the literature the issue of vorticity is nearly always dropped before actual calculations are done, and models which completely lack the ability to represent vorticity or virialization (such as LTB) are usually used for the estimates of backreaction-related effects of inhomogeneities on the observed cosmic expansion.

As Hirata & Seljak (2005) note, the absence of vorticity plays a key role in all of the "no-go" theorems used to prove the impossibility of accelerated expansion in the absence of some kind of SEC-violating Dark Energy. And therefore, due to the crucial importance of vorticity (and thus a substantially nonzero ω^2) in structure formation for all of the virialized inhomogeneities pervading the universe, we can conclude that *all acceleration "no-go" theorems are cosmologically inapplicable.* A smoothly-inhomogeneous cosmology with no Dark Energy, but with the overall effects of pervasive virialization, can indeed (as we will see) be able to produce enough of a backreaction to create (at the very least) an apparent acceleration, if not a full-fledged volumetric ('real') acceleration. Vorticity and velocity dispersion effects are not in fact a small-scale player in the physical universe, but represent the major player (along with gravity), and the only question that remains is to figure out how to properly incorporate them.

The difficulties involved in explicitly including vorticity within cosmological models are very real, however, and it would be a highly nontrivial problem to try to precisely compute the detailed time evolution of (and volume expansion from) a realistically virializing structure. Fortunately though, as we contend here, there is no need to solve this exact problem in order to get a good approximation of the averaged effects of backreaction upon the observed expansion rate. Instead, we can exploit the fact that both the beginning state (nearly perfectly smooth FRW matter) and the ending state (a reasonably random distribution of discrete Newtonian mass concentrations) are extremely simple, making the net results of the phase transition to inhomogeneity very straightforward to estimate. Only the general evolution of how much of the cosmic matter has completed the phase transition to clumpiness as a function of cosmic time is important, not the detailed time dependence of the virialization of any individual object; and for this paper we will use a set of reasonable heuristic models to empirically determine which ones are best suited for fitting the observational data. This phenomenological formalism will be motivated and presented below, in Section 3.

2.3. Newtonian-Level Backreaction: Not Suppressed, Not Small, and Not Slow

There remain debatable issues regarding the possible *magnitude* of such backreaction effects, however, given the fact that we continue to rely here upon (individually) Newtonian perturbations, and do not explicitly recruit the effects of nonlinear-strength gravitational fields. Powerful theoretical arguments have been advanced by previous researchers, claiming that cosmological backreaction from Newtonian-level perturbations must necessarily be very small, if not actually suppressed to *zero*, in a practical sense. Given that the development of Newtonian perturbation terms will provide positive contributions to the spatial metric components (to g_{rr}, in particular, for the spherically-symmetric case), it is unclear why new volume expansion is not considered an inevitable result of those newly-developing Newtonian perturbations; but it is this assumed restriction for backreaction, apparently requiring one to appeal to higher-order gravitational terms, which is what has forced researchers to resort to desperate measures in order to achieve a significant backreaction. This author's original attempt at backreaction (Bochner 2007a), for example, postulated a toy model utilizing black holes, in order to generate a strong enough effect to make a difference in the cosmic expansion (a clearly unworkable model for several reasons, but no alternative was obvious at the time). Using another exotic approach, other researchers have attempted to recruit super-Hubble-sized density perturbations – first on their own (e.g., Kolb et al. 2005a,b), and then in tandem with sub-Hubble terms (Kolb et al. 2006) – in order to create a usefully large inhomogeneity-induced backreaction, in spite of the causality-related difficulties (Hirata & Seljak 2005) of trying to produce physically measurable effects from perturbation modes larger than the observable horizon. Kolb et al. (2008) find it necessary to go to the extent of arguing that a perturbed conformal Newtonian metric is not even an appropriate description for a universe observationally well-described by a ΛCDM model, when considering deviations from an Einstein-de Sitter dust model as the unperturbed background. It is clear, therefore, that the permissibility of a Newtonian description of inhomogeneous perturbations is commonly viewed as an insurmountable impediment for backreaction. We must therefore examine the reasons for this belief, and show how it comes about from an unwarranted oversimplification of cosmological physics.

From the extensive work of Buchert and collaborators (e.g., Buchert & Ehlers 1997; Buchert et al. 2000), it is well known that the entire Newtonian backreaction, Q_N, can be expressed mathematically as a total divergence: $Q_N \equiv Q_{Div} \equiv \nabla \cdot \boldsymbol{q_N}$. We know from the divergence theorem (e.g., Jackson 1975) that for any such function integrated within a volume V (with boundary δV), we can relate the volume-integral of Q_N to the boundary-normal surface integral of $\boldsymbol{q_N}$: $\int_V Q_N = \oint_{\delta V} \boldsymbol{q_N} \cdot \boldsymbol{n}$. This surface integral will vanish if the universe is topologically closed (i.e., if there *is no* surface), or if periodic boundary

conditions are assumed (as is typical for large-scale Newtonian cosmological simulations – e.g., Springel et al. (2005)), meaning that the total integrated Newtonian backreaction ($\int_V Q_N$) would consequently have to vanish. But even if it does not completely vanish, Kolb et al. (2006) argue that the overall effect will in any case be tiny, because the surface term involves the peculiar velocity $\boldsymbol{u}$, which should be small in Newtonian situations; and perhaps another limiting factor here is that an integral over a surface of radius R will only increase as $\sim R^2$, so that the volume average will go like $\sim R^2/R^3 \propto 1/R$, becoming negligibly small for arbitrarily large R. Thus the ability to represent Newtonian (i.e., gravitationally linear) backreaction as a total divergence, if valid, would act as a strong suppression of the magnitude of the effect.

It is interesting, at this point, to consider the basic equations at the heart of the formalism used for placing these limits upon the Newtonian-level backreaction – specifically, Equations 1a-1d in Buchert & Ehlers (1997) – and to compare them with Maxwell's Equations (along with the Lorentz force law and continuity equation) for electromagnetism (e.g., Jackson 1975). A careful examination of these expressions shows that for the former, something is missing: the magnetic fields. The Buchert formalism, and the various theorems derived from it, have all been done having set what one may call the "gravitomagnetic" fields (e.g., Mashoon et al. 2001) to zero. Now, the neglect of the 'magnetic' fields in electrodynamics (or in Newtonian gravity) happens to be a very significant simplification, as is obvious from a quick examination of Ampère's Law, $\nabla \times \boldsymbol{B} = (4\pi/c)\boldsymbol{J} + (1/c)\partial \boldsymbol{E}/\partial t$. If $\boldsymbol{B}$ is set to zero, then the time rate of change of $\boldsymbol{E}$ in a region is solely dependent upon the local current/momentum flow, $\boldsymbol{J}$, inside that region. In other words (say, for gravitation), the contributions to the local Newtonian gravitational potential and fields by distant masses outside the local volume are *completely frozen-in*, with any possible causality-driven dynamics being entirely suspended. This semi-static formulation (i.e., considering only the motions of *local* masses) is entirely in accord with the common view (Buchert et al. 2000) that: "In the Newtonian approximation the expansion of a domain is influenced by the inhomogeneities inside the domain." The main thrust of our arguments in this paper is to point out that this view is unacceptably incomplete, and in fact we rejected the Swiss-Cheese approach earlier on this very same basis, that it is physically unrealistic to bar different regions of spacetime from communicating with one another. While such an approximation may be reasonable according to what researchers usually think of as a 'Newtonian' approximation, it is certainly not compatible with the dictates of special relativity and cosmological causality.

For an alternate way of expressing Buchert-like restrictions on backreaction, consider the arguments of Ishibashi & Wald (2006), in regards to the following, "Newtonianly-Perturbed FLRW" metric:

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Phi)\gamma_{ij}dx^i dx^j \; . \tag{3}$$

In analyzing this metric, they adopt the various 'Newtonian' conditions for the potential function, Φ:

$$|\Phi| \ll 1 \, , \tag{4a}$$

$$|\partial\Phi/\partial t|^2 \ll \frac{1}{a^2}D^i\Phi D_i\Phi \, , \tag{4b}$$

$$(D^i\Phi D_i\Phi)^2 \ll (D^i D^j\Phi)D_i D_j\Phi \, . \tag{4c}$$

They point out that even in the regions of very large local density variations, $\delta\rho/\rho$, the metric perturbation Φ due to these spatially-varying matter concentrations should always remain small everywhere (with the exception of extreme regions, such as near black holes); and that given these conditions, the nonlinear corrections to the expansion will remain small. The linear effects are small by hypothesis, and Ishibashi & Wald (2006) show that the use of Equations 4a-c implies that the remaining dominant backreaction term for Φ satisfies an expanding-universe version of the Poisson equation, basically reducing it to a total divergence and thus subject to Buchert suppression.

Now, one may concede that spatial variations in Φ are indeed small as per the arguments of Ishibashi & Wald (2006); and if the temporal growth in Φ were also small due to Equation 4b, then the perturbation potential Φ (being negligible in the early universe) would thus have remained very small virtually everywhere and at all times in cosmic history to this point, so that little backreaction could ever have been generated from it. But even dropping the idea of large spatial variations – or of *any* spatial variations, which we do completely neglect in the case of a smoothly-inhomogeneous universe – why should one assume that Equation 4b is a valid approximation?

In computing the evolution of structure formation, one typically utilizes the Poisson equation, $\nabla^2\Phi = 4\pi G\rho$ (for now ignoring modifications to it needed for representing the cosmic expansion). It would of course be okay to follow this approach (as in the Buchert formalism) if it were safe to ignore the gravitomagnetic fields, since the "gravitoelectric" fields thus become curl-free. But this is not the rigorously correct formula to use, because the Poisson equation is *not causal* – its (unphysical) solution is the integral of the *instantaneous* Coulomb potential (at that point) due to the charge/mass density distributed throughout all of space, integrated out to infinity. While such an approach may be appropriate for considering a single, localized physical system embedded in an asymptotically empty universe, this picture makes no sense for a system embedded in an essentially infinite universe with important nonequilibrium processes going on everywhere, for which the expanding observational horizon is always bringing new and important information (and perturbative forces) in towards the observation point of that system.

For a causality-respecting approach, one must instead use the full wave equation (for

special-relativistically-consistent potential function Φ_{SR}, in analogy with the electric potential in the Lorentz gauge, rather than the Coulomb gauge):

$$\nabla^2 \Phi_{\mathrm{SR}} - \frac{1}{c^2}\frac{\partial^2 \Phi_{\mathrm{SR}}}{\partial t^2} = 4\pi G \rho \; . \tag{5}$$

Now, the usual impulse is to immediately drop the extra term in Equation 5, involving $\partial^2 \Phi_{\mathrm{SR}}/\partial t^2$ (equivalent to dropping the gravitomagnetic terms), because of its resulting prefactor of v^2/c^2; this factor would seem to make it very small given the (reasonable) assumption of nonrelativistic speeds for most matter flows, and thus (assumedly) ensuring it to negligible compared to the spatial variations term in any backreaction calculation. But this thinking is based only upon considerations of individual Fourier perturbation modes, not on the overall causal behavior of information flow in the structure-forming universe. If we instead *keep* all terms, and solve Equation 5 as-is, then one gets (Jackson 1975, eq. 6.69):

$$\Phi_{\mathrm{SR}}(\mathbf{x}, t) = -G \int^{\infty} \frac{[\rho(\mathbf{x}', t)]_{\mathrm{ret}}}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \; , \tag{6}$$

where the bracketed numerator is always evaluated at the *retarded time*, $t' = t - |\mathbf{x} - \mathbf{x}'|/c$. It is this retarded-time condition which restores causality, allowing different regions of the universe to communicate with (and gravitationally perturb) one another; and which provides the escape route from the Buchert suppression of Newtonian-level backreaction, because the backreaction resulting from this integrated perturbation potential is *not* expressible as a total divergence. We will refer to this propagation of gravitational perturbation information between distant (though communicating) regions as "causal updating".

Now certainly, the factor of v^2/c^2 multiplying such causal updating contributions is typically quite small; but there is a crucial difference between "small" and "suppressed". Many individually small contributions can be added together to produce a large overall result, if one has enough of them. And the perturbative contributions to the metric at a specific location are supplied by *every* virializing structure in the universe that is within the causal horizon of that location. Consider that the gravitational potential (and thus the Newtonian-level perturbation effects) of some particular mass at distance r decreases only like $\sim 1/r$, while the number of such masses within a spherical shell at that distance goes like $\sim r^2$. Summing up the small but nonzero perturbing contributions of all 'remote' virializing clumps, in their effects upon any particular observer's location, and integrating out to infinity, one gets a total effect $\propto \int^{\infty}(r^2)(1/r)\,dr = \int^{\infty} r\,dr = \infty$! Noting once again that each individual contribution is from a strictly *Newtonian* gravitational potential, we see that the net effect is not only non-small, but in fact it is infinite (in this very simplified picture), rendering any factors of v^2/c^2 moot. The only effects which actually rein in the total integrated perturbation to make it finite in the real universe, are the finite causal horizon out

to which an observer can 'see' perturbations (with more distant regions of space obviously looking younger and less clumped, due to larger look-back times); and the cosmic (Hubble) expansion, which dilutes it by continually carrying the virialized, clumped structures farther and farther away from the given observation point. (This situation is reminiscent of the problem, and the solution, of Olbers' Paradox (Weinberg 1972); except that the 'infinity' is even stronger here, since the Newtonian gravitational potential only fades with distance as $\propto 1/r$, rather than $\propto 1/r^2$, as with light intensity.)

Though this infinity is not physically realized, what the infinite behavior of this simple integral actually does alert us to, is that the total perturbation may eventually get very large – ultimately defying even the assumed condition $|\Phi| \ll 1$ from Equation 4a – meaning that the entire method of summing Newtonian potentials and using a Newtonianly-perturbed metric will break down completely as the backreaction grows strong enough to become general-relativistically nonlinear. Thus the summed effect of innumerable, individually-Newtonian perturbations is no longer 'Newtonian' in total. (We must therefore watch the calculated values of Φ_{SR} produced in our numerically simulated models very carefully, recognizing the loss of accuracy of our formalism when this quantity approaches a value of order unity.) But even being finite and (in many cases) Newtonian in total, it is clear that the impact of distant shells of clumpy material upon the observer's metric can strongly outweigh the effects of inhomogeneities located in the actual vicinity of the observer.

To the extent that a linearized-gravity approximation is valid, our smoothly-inhomogeneous approach therefore works by computing a time dependent, spatially averaged perturbation potential $\Phi_{\mathrm{SR}}(t)$, which we do *not* assume is slowly-evolving, as is (inappropriately) assumed for perturbation theory treatments via Equation 4b. Realizing that the summed effect of a spherical shell of perturbing clumps at distance r actually *increases* as $\sim r$, this means that the dominant backreaction contributions will be delivered to an observer by masses at the *farthest* possible distances out to which that observer can still see significant matter inhomogeneities; while past that point, the retarded-time observations are from a look-back time too early in the cosmic history for substantial structure formation to have yet occurred, thus making the resulting integral finite.

Considering the fact that the key metric perturbation function, $\Phi_{\mathrm{SR}}(t)$, is predominantly affected by perturbation information coming in from distant locations, the retarded-time condition of an integrated formula like Equation 6 (suitably modified for cosmological calculations) implicitly gives it the ability (contrary to the argument in Gromes (2011)) to exhibit relativistic behavior in what would otherwise appear to be nonrelativistic situations. To see this, consider the following argument: suppose that there is some approximate transition time, t_{clump}, when the universe evolves (not necessarily instantaneously) from being

essentially smooth to essentially clumpy. Now at any later time, $t > t_{\text{clump}}$, the physical radius out to where an observer can 'see' this transitional epoch is given (again ignoring cosmic expansion here, for simplicity) by $r_{\text{clump}} \sim c(t - t_{\text{clump}})$, a radius which increases over time, expanding outward from the observation point (say, Earth) at the speed of light. The backreaction effects get bigger and bigger (due to increasing shell size and thus a greater number of contributing perturbations) as this outgoing "wave of observed clumpiness" expands outward from us at c, thus being fundamentally relativistic in nature and defying the assumption of small $\partial \Phi / \partial t$ [4].

Thus we see that the potential function Φ_{SR} can be fully (special-)relativistic in character, violating the assumed conditions placed upon 'Newtonian' perturbations by Ishibashi & Wald (2006), *even if none of the matter in the universe is actually moving at relativistic speeds.* This unexpected effect is due to the fact that cosmological systems (unlike anything else) are infinite in size, meaning that the inherently relativistic act of causal observation will end up encompassing volumes which are incomparably vast, causing even small effects – which never cancel out in gravitation, unlike in electrodynamics – to sum together to produce a dominant overall influence. Perhaps surprisingly, therefore, the apparent acceleration of the universe that we observe is not caused by any individually powerful or gravitationally exceptional specific objects or regions, but rather from the summed influences of the weak-gravity Newtonian tails of innumerable mass concentrations, imposing their combined effects upon us from extraordinarily large distances.

In this approach, it is clear that we are advocating a diametrically-opposed view from that of other authors who have sought to define a "finite infinity" (e.g., Cox 2007), representing a very limited boundary from within which significant effects upon some specified local volume can have ever arrived. Such a limitation would restrict the history of 'important' interactions to be within an effective "matter horizon" (Ellis & Stoeger 2009) of only a few Mpc in size, delineated by the timelike world lines traveled by pressure-free matter due to scalar perturbations. But though they attempt to make the case that effects from outside of this matter horizon (yet within the fundamentally causal "particle horizon") are generally very small, the chief effects actually estimated by Cox (2007) include the Weyl terms regarding geodesic deviation ($\propto 1/r^3$), and the power radiated in terms of the Bondi news function ($\propto 1/r^2$), both of which are vastly smaller at cosmological distances than the *amplitude* terms ($\propto 1/r$) perturbing the metric itself, which are what we study here. Notably, it has been shown by Ludvigsen (1989) that even an arbitrarily small energy flux due to

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[4]As an analogy, consider the contact point between the two blades of a very long pair of scissors. The rate at which this contact point moves outward from the central pivot does not represent the physical motion of any real object, and hence is not limited by the speed of the material in the blades as they come together.

gravitational waves can result in a finite amount of "geodesic deviation" (actually, long-term positional displacements of observers due to permanent metric changes) in the very distant radiation zone. Cox (2007) in fact concedes that for galaxies (or clusters, etc.), "the prospect of adequately treating such a diffuse body as isolated is doomed", due to such very-long-wavelength gravitational radiation (incidentally demonstrating once again the inadequacy of the Swiss-Cheese approach).

The point is that even such small metric alterations can become significant (even dominant) when summed over the very many contributing sources within one's causal horizon. Perhaps the most concrete way of demonstrating the validity of this idea, is to point out that Cox (2007) and Ellis & Stoeger (2009) both acknowledge the Great Attractor/Shapley Concentration as a likely source of our bulk flow – i.e., as the completely dominant influence upon the motion of our entire neighborhood of galaxy clusters – despite the clearly contradictory fact (for their claims) that those structures which control our bulk motion are exceedingly far beyond any reasonable estimate of our so-called matter horizon. Thus the assertion in Ellis & Stoeger (2009) that, "the strengths of any other possible long distance influences... gravitational waves, or electric Weyl tensor components from sources outside that region – are insignificant compared to local effects," is simply incorrect.

Despite the incontestable importance of those very distant mass concentrations in determining such 'local' motions (and thus necessarily, our local metric), there are very straightforward calculations using gravitational perturbation theory which are commonly believed to contradict all assertions of the significance of backreaction in general (and of nonlocally-acting causal backreaction in particular). The problem is that despite the standard (almost reflexive) tendency for experts to resort to perturbation theory when attempting to evaluate the effects of cosmic structure formation, the fact is that perturbation theory is singularly ill-equipped for dealing with the most important perturbative effects in an unbounded system like the (effectively) infinite universe. The central difficulty is that perturbation theory cannot make any predictions unless one pre-specifies which approximations to make, and thus which terms to drop. For example, Kolb et al. (2005a) neglect information-carrying tensor modes generated by nonlinear scalar perturbations (i.e., virializing masses) in the final expressions for their analysis (as well as assuming irrotational dust, thus neglecting vorticity), which leaves one with scalar perturbations only (vector modes are also dropped), thus once again limiting the calculation to effects coming from within the inappropriately small (as we have seen) matter horizon. Alternatively, Räsänen (2012) does comment upon the importance of the propagating degrees of freedom due to the "magnetic" gravitational component $H_{\alpha\beta}$ (the same thing as our "gravitomagnetic" fields discussed above), and notes the difficulty of studying backreaction using a post-Newtonian scheme without them, given that the usual assumption of a finite and isolated system in such calculations is an inap-

propriate condition to adopt in a cosmological setting. Nevertheless, Räsänen (2012) then effectively eliminates such terms by re-affirming the approximation that the time derivatives of metric perturbations are small – similar to the approximation above in Equation 4b, which we have labored here to refute on the basis of supporting the causal flow of gravitational information. Now obviously, if one removes all "causal" aspects from "causal backreaction", then nothing will be left, and it is unsurprising for one to then find that backreaction fails to work, given that the principal physics responsible for it has once again been set to zero.

The essential problem with perturbation theory in regards to these issues relates to its basic program of singling out the 'important' physics by labeling the amplitude of each term as "large" or "small", and then dropping the small terms in order to focus upon the large ones. But this basic procedure is conceptually flawed in the case of causal backreaction, which deals with propagating modes, since such modes can bring in gravitational perturbation information to a local observer from vast spatial volumes, so that a term with a 'small amplitude' can actually produce an enormous overall effect. For example, both Räsänen (2012) (as noted above) and Gromes (2011) assume that time derivatives of metric perturbations are small, leading to the conclusion that velocity-dependent terms can safely be neglected; and Gromes (2011) specifically uses this point as a primary argument for dismissing the effects of causal backreaction entirely[5]. But conflating "small" with "negligible" is a continuing error in perturbation theory analyses of cosmological evolution, since the *size* of these effects, in terms of amplitude, effectively does not matter: it is practically irrelevant how small the time-derivative terms may be, when one realizes that such effects are being cumulatively contributed by *all* of the matter within a causal horizon that may be billions of light-years in extent, thus multiplying that amplitude by an amount of mass easily large enough to overcome its inherent smallness. Furthermore, even if the amplitudes of the relevant perturbative terms for causal backreaction were somehow 'magically' made even smaller than they naturally are, this still would not shut off the causal backreaction effect, but merely postpone it to begin a little bit later, since a somewhat larger causal horizon of self-stabilizing inhomogeneities would then be needed to supply a large enough integrated effect to cause an apparent acceleration. In that sense, one could *never* make the amplitudes of those perturbation terms small enough to avoid the eventual dominance of causal backreaction, since a big enough causal horizon (containing a sufficiently large amount of

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[5]Gromes (2011) also uses an argument referring to their "optimal gauge", in justifying the neglect of causal backreaction; but as they themselves note, it can be a tricky thing to choose the right gauge when trying to connect theoretical expressions to real cosmological observables, and a different gauge would lead to a different amount of the kind of backreaction effect which we consider here. Thus the permissibility of choosing gauge conditions that set components of the backreaction to zero, a procedure which they engage in, is a highly nontrivial matter.

virializing mass) can always be attained after a sufficiently long period of time, which when multiplied by even the smallest amplitude, would eventually produce a term of order unity that then proceeds to dominate the cosmic evolution. Thus for causal backreaction in an effectively infinite universe, *there is no such thing as "negligible"*.

3. DEVELOPING A PHENOMENOLOGICAL FORMALISM

3.1. Observational Horizons and the Evolving Perturbation Potential

As long as the universe remains homogeneous and isotropic enough to be considered at least smoothly-inhomogeneous, then as pointed out at the end of Section 2.2, the net result of virializing structure formation is quite simple: it consists of the replacement of continuous FRW-distributed matter with a discrete collection of stabilized clumps, distributed (one assumes) fairly randomly. Considering the metric perturbation effects upon a specific volume $\mathbf{V}$, a given stabilizing structure $\mathbf{S}$ will only produce nonzero backreaction within $\mathbf{V}$ for as long as new gravitational information is propagating from the structure to and through that volume. Once the contributing gravitational potential of the final, stabilized state of $\mathbf{S}$ has had the opportunity to completely propagate causally throughout the entirety of $\mathbf{V}$, then as per the Buchert limitations on Newtonian backreaction discussed above, the backreaction in $\mathbf{V}$ due to $\mathbf{S}$ will be over. This variety of backreaction, due to a set of individually-Newtonian virializing structures, is therefore just a one-shot deal, unlike the kind of self-sustaining acceleration possible for truly general relativistic (i.e., nonlinear) backreaction effects. On the other hand, a never-ending collection of such one-shot contributions can by itself produce a persisting – and even growing – backreaction effect within $\mathbf{V}$, as structures at ever-greater distances come into the causal horizon of $\mathbf{V}$ over time.

We wish to come up with a simple representation of how the metric within $\mathbf{V}$ is altered by such one-shot perturbing contributions. An attempt by other researchers to estimate the total backreaction effects for a somewhat similar situation – for that of an infinite lattice of compact, static masses – was done by Gruzinov et al. (2006), which once again found the main backreaction effect to be small, of order $O(H^2 l^2/c^2)$, where l is the lattice size. But this result is only obtained as the leading-order backreaction term after the subtraction of an apparently divergent term due to the classical Newtonian gravitational energy of the masses, a 'divergence' which they assumed to be unphysical and thus removed via a "bare mass" renormalization step. But we have already seen this apparently infinite term above in Section 2.3, where we argued that the term was quite real, and rendered finite by the finite amount of time that it takes for inhomogeneities to initially form, and (more importantly) by the finite cosmic horizon out to which one can see those inhomogeneities. In a calcula-

tion using an eternal lattice of static structures, however, such a term would appear to be genuinely infinite and unphysical, and would thus unfortunately be dropped.

In order to figure out how to properly implement Newtonian-level backreaction with causality in the absence of an exact cosmological solution, we do a little thought experiment. Consider our local volume $\mathbf{V}$ to be a homogeneous spherical region cut out of a matter-dominated, nearly perfectly homogeneous early universe. Now, it is well known (Weinberg 1972, pp. 474-475) that the expansion evolution (i.e., the Friedmann equation) for $\mathbf{V}$ can be derived – using nonrelativistic Newtonian equations, in fact – without reference to anything outside of that sphere. Barring perturbations, the Friedmann evolution of $\mathbf{V}$ is determined completely internally, and all matter outside of it is gravitationally unmeasurable, as if the universe outside of $\mathbf{V}$ did not even exist. So let us *remove* it, leaving an infinite vacuum surrounding our spherical region $\mathbf{V}$.

This perfect isolation cannot last forever, of course, since later on, distant stabilized structures will form which certainly do exert real effects upon our local volume by becoming attractors which pull mass in towards themselves from all directions, including from within $\mathbf{V}$. These gravitational effects, unknown to $\mathbf{V}$ before that time, will seem to appear as new from the 'empty' region outside of it, as if such clumped structures were brought in from infinity at some recent time during the structure formation epoch, to only then begin gravitationally affecting $\mathbf{V}$.

In most cases, one concerns oneself solely with the spatial *gradients* of the gravitational potential, Φ, caused by the distant perturbations; and so one is able to study overdensities and voids, bulk flows, and the like, even perhaps to the extent of trying to achieve cosmological backreaction with them. But we have completely dropped those spatial gradients in our adoption of a smoothly-inhomogeneous version of the Cosmological Principle, since a randomly-distributed collection of inhomogeneities would exert forces in all directions, largely (if not perfectly) canceling each other out.

But what does *not* cancel out, regardless of how the perturbations outside of $\mathbf{V}$ are distributed – because it is a non-directional scalar, rather than a vector – is their combined contribution to the overall level of Φ; that is, its actual magnitude, which does not matter in true Newtonian physics (only differences in potential do), but which does matter in general relativity. The contribution by all clumped masses to Φ within $\mathbf{V}$ (i.e., $\Phi_{\mathbf{V}}$) will always be negative (as gravitational energy is negative), thus adding together constructively and reinforcing their total effect upon our local volume, even for perturbing masses located on opposite sides of it. Furthermore, negative contributions to $\Phi_{\mathbf{V}}$ due to overdensities will not be canceled out by positive contributions due to underdensities and voids, because the situation is not symmetrical (and in fact is biased towards volume creation): underdensities

always keep expanding, and at a faster rate than the cosmic average; but overdensities do *not* keep shrinking, because they eventually stabilize themselves through vorticity and virial-ization. And these effects go far beyond the specific regions containing the clumped masses themselves, since by adding a nonzero net contribution to $\Phi_\mathbf{V}$, these exterior perturbations affect both the flow of time and the actual physical volume (consider the spatial metric terms of Equation 3) within our local region of space. The fact that the effects of such distant per-turbations are individually small ($\propto 1/r$) does not matter when one can sum the combined effects of all of the masses in spherical shells out to very large r.

Our phenomenological approach is therefore one in which we model the inhomogeneity-perturbed evolution of $\mathbf{V}$ with a metric that contains the individual Newtonian perturbations to the potential $\Phi_\mathbf{V}(t)$ from all clumped, virialized structures outside of $\mathbf{V}$ that have been causally 'seen' within $\mathbf{V}$ by time t, superposed *on top of* the background Friedmann expansion of $\mathbf{V}$. Now, one may object that material outside of $\mathbf{V}$ actually falls within the observational horizon of $\mathbf{V}$ long before it became inhomogeneous, yet had no effect upon $\mathbf{V}$ whatsoever at that time; and thus such mass should not provide any *new* contributions to $\Phi_\mathbf{V}$ at a time later on, simply because it has spatially redistributed itself from being smooth into being clumpy. But our reply would be that each individual structure – only *after* it has clustered and virialized itself – exerts its own individual pull upon the mass inside $\mathbf{V}$, causing a peculiar-motion acceleration of the mass inside $\mathbf{V}$ towards itself, and that this is a new force (representing new "gravitational knowledge") not seen before within $\mathbf{V}$. But one cannot feel a gravitational *force*, unless one simultaneously feels a gravitational *potential*; and so this perturbing potential must be new within $\mathbf{V}$ as well, approaching and entering $\mathbf{V}$ in causal fashion from this clumped object (and from all others) as they develop over time, everywhere in the surrounding universe.

Now from the perspective of a general-relativist, the natural impulse would be to declare this entire procedure non-gauge-invariant, and then simply transform $\Phi_\mathbf{V}(t)$ away via a new time coordinate. But we assert here that this would be a physically improper procedure, since the act of making a cosmological observation is itself not a gauge invariant process. General relativity only guarantees a physical invariance under *local* coordinate transformations; but cosmological measurements are manifestly nonlocal – most importantly here in regards to the time coordinate – given that the Hubble curves used to demonstrate the existence of a cosmic acceleration are not measurements of a metric quantity at a single instant of time (or a single location in space), but are produced by *integrating* the motion of a light ray over its past light cone, an integration over timescales that are by design much longer than what could possibly be considered a 'local' range of t, in order to detect evolution of the cosmic expansion rate $a(t)$. In cosmology, therefore, the time coordinate cannot be transformed to any other $t'(t)$ time function with impunity, since the original t coordinate has a unique physical meaning:

it governs the rate of cosmic expansion (e.g., $a(t) \propto t^{2/3}$ during matter domination), which is measurable in many ways. This cosmic time t thus serves as an absolute clock which we can use as a reference for comparison against the rate of local physical processes – for example, measuring how fast light rays travel through the universe, versus how long it takes the cosmic scale factor to double in size. Therefore, we claim that this $\Phi_{\mathbf{V}}(t)$ function – which affects local physics such as light-ray propagation, but (ironically, for backreaction) does not alter the cosmic expansion rate function $a(t)$ – encapsulates real physics that produces observable consequences, and as such cannot be legitimately transformed away.

The overall picture presented here is clearly a very heuristic one, and the growing perturbation potential $\Phi_{\mathbf{V}}(t)$ would of course not represent the *literal* metric of the universe; rather, it is simply one that makes sense as a qualitative shorthand for the essential backreaction effects generated by structure formation throughout the cosmos, as they act upon any particular patch of space. What is really happening, in a more complete physical sense, is that collapsing overdensities stabilize themselves and halt their collapse by concentrating their local vorticity; this concentrated vorticity leads to real, extra volume expansion, representable (in the final state) at great distances by the tail of a Newtonian perturbation potential to the background FRW metric; and this Newtonian tail propagates continually outward into space by inducing inward mass flows towards the virialized object from farther and farther distances, as time passes. The total perturbation at any location in space (which will be independent of position, assuming similar structure formation rates everywhere) will then be the combined effects of innumerable Newtonian tails of this type, coming in all the way from cosmological distances, and from every direction. All that our thought experiment has allowed us to do is to show that it is reasonable to represent all of these complex physical processes in a very simple way, as a cosmologically uniform perturbation potential $\Phi_{\mathbf{V}}(t)$ superposed on top of the original FRW expansion, where the potential grows in time as gravitational information about virialized structures flows in from an ever-expanding "inhomogeneity horizon". It is the changing absolute level of this causally-developing gravitational potential function (referred to earlier as $\Phi_{\mathrm{SR}}(t)$, to emphasize special-relativistic causality) that we argue is the *real* main effect of backreaction; and we will show it to be sufficient for producing an observed acceleration completely on its own, in a smoothly-inhomogeneous context that does not require any other inhomogeneity-related effects.

3.2. Inhomogeneity Evolution and Causal Integration for Metric Updating

In order to derive an expression for the evolving perturbation function $\Phi_{\mathrm{SR}}(t)$, we start by considering the phase transition itself, during which an extremely symmetrical and smooth

("unclumped") state during the early universe has gradually evolved to an almost entirely "clumped" state today. During the transition, most of the matter in the universe is converted from something close to a completely smooth perfect fluid (say, pressureless dust), to a discrete but infinite collection of randomly-distributed clumps of various sizes. We must define a physical quantity as a measure of 'cosmic clumpiness', to represent the extent to which this phase transition has proceeded to completion. One could perhaps use the ratio of the typical size of a stabilized mass clump divided by the typical distance between clumps as a symmetry-breaking parameter; or alternatively, one could define an order parameter, given as the fraction of cosmic mass in the clumped (i.e., virially-stabilized) state, versus that remaining in the smooth (i.e., freely-expanding FRW) state. We choose the latter approach; and as the order parameter grows large, this indicates the impending breakdown of the matter-dominated FRW evolution of a pure-dust model, and the growing relevance of inhomogeneities.

To phenomenologically model the evolution of clustering, we therefore define a "clumping function", $\Psi(t)$, defined (as a fraction of the total density) over the range from $\Psi(t) = 0$ (perfectly smooth matter), to $\Psi(t) = 1$ (everything clumped). Note here that we are assuming a spatially flat background universe everywhere in our calculations; and given that our formalism is specifically designed to eliminate the need for Dark Energy, this implies that all of the cosmic contents are treated as pressureless dust (for convenience ignoring relativistic matter and energy, which do exist but in lesser amounts), and that such dust adds up to $\Omega_M \equiv 1$. (This latter condition is only in reference to the unperturbed background, however, given that the apparent observational value of Ω_M will evolve to become quite different from unity, as we will see.) We will not be rigorously deriving $\Psi(t)$ from first principles, but will rather (as described in later sections) be utilizing a variety of functional forms and parameters motivated by basic physical principles and measurements, and then empirically evaluate their performance in light of cosmological observations.

Obtaining a metric which can explicitly represent the full dynamics of a self-stabilizing structure, complete with vorticity and virialization, is an enormously complex task; it is so challenging, in fact, that we have already seen how exact solutions (like Swiss-Cheese models) and more general methods (like the Buchert formalism) have both failed in this task, due to the overly-restrictive assumptions that they were forced to adopt in order to simplify the situation enough to provide a tractable analysis. We therefore seek a practical way in which the key backreaction effects of clustering (described by clumping evolution function $\Psi(t)$) can be simply but effectively modeled, given our smoothly-inhomogeneous formalism with causal updating.

We must begin by considering a single clumped mass (representing a virialized, self-

stabilized inhomogeneity) that is embedded within the expanding universe. It is actually quite easy to embed a single clump within an empty, coasting universe by considering the "Expanding Minkowski Universe" (Robertson & Noonan 1968) with $a(t) \propto t$; this metric is mathematically equivalent to the (static) Minkowski spacetime of special relativity, and can be transformed to it via the transformation: $t' = t\sqrt{1 + r^2}$, $r' = tr$ (Misner, et al. 1973, p. 743, using (t', r') instead of their (t, χ) coordinates). The inverse transformation is readily obtainable as: $t = \sqrt{(t')^2 - (r')^2}$, $r = r'/\sqrt{(t')^2 - (r')^2}$ (noting that the entire coasting universe is covered by just the $t' \geq r'$ portion of the Minkowski metric). By using the simple trick of applying this inverse coordinate transformation to the static Schwarzschild metric[6] of a Black Hole (e.g., Weinberg 1972), rather than to pure Minkowski space, instead of producing a completely empty coasting universe we now obtain an (essentially) coasting universe which possesses a clumped mass at the origin. There is the drawback that such a cosmology is still empty everywhere outside the central mass, and thus has a negative global curvature and cannot represent a matter-filled cosmology; but a few appropriate approximations and ansatzes can readily convert this metric into a spatially flat, matter-dominated metric with a clumped mass at the origin.

If one objects to such extrapolations, then the exact same resulting metric can be obtained by linearizing the McVittie solution (McVittie 1933), as can be seen from the perturbed FRW expression given in Kaloper et al. (2010). One thus gets, for the Newtonian approximation of a single clumped object of mass M, embedded at the origin ($r = 0$) in an expanding, spatially flat, matter-dominated (MD) universe:

$$ds^2 \approx -c^2[1 + (2/c^2)\Phi(t)]dt^2 + [a_{\mathrm{MD}}(t)]^2[1 - (2/c^2)\Phi(t)]dr^2 + [a_{\mathrm{MD}}(t)]^2 r^2[d\theta^2 + \sin^2\theta d\phi^2] \,, \quad (7)$$

where $\Phi(t) \equiv \{-GM/[a_{\mathrm{MD}}(t)r]\}$, and $a_{\mathrm{MD}}(t) \propto t^{2/3}$ is the unperturbed MD scale factor evolution function. Note that this McVittie solution actually includes no mass accretion – i.e., M in the above metric is a constant – even though such accretion would certainly be expected to occur in a realistic cosmology due to inflows onto any overdensity embedded within the matter-filled universe (and this was indeed one of our main arguments earlier for disregarding the Swiss-Cheese limits on backreaction). In any case, this "M" does not simply represent the total quantity of mass (i.e., conserved 'dust') within some specified volume, but rather embodies the amount of *clumped and virialized* mass in that volume (which grows

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[6]Given the key role of spinning and vorticity in our discussion of backreaction, one might expect the use of the Kerr metric here; but the vast majority of mass in the universe does not possess relativistic amounts of angular momentum – merely a sufficient degree of it, acting continuously for a cosmologically-long time, to ensure that the object remains stabilized. Thus we treat the basic existence of persistently-stabilized Schwarzschild-like point masses as the biggest perturbation here, without needing to include the further metric perturbation which results from their actual rotation.

in time as a reaction to inflows onto overdensities); and so our phenomenological model will therefore effectively assume a time-dependent expression, $M(t)$, in the above metric, where this time dependence is derived from the clumping evolution function $\Psi(t)$, as we now work out in detail.

Assuming that various mass concentrations M_i will be randomly and (sufficiently) evenly distributed throughout the universe, one must integrate over distance (from a particular observer's location) for the various clumps, and one must also *angle-average* over them to get the metric for the combined effects that would represent an averaged ds^2 typical for that observer experiencing displacements in any given direction. (Angle-averaging the effects of an ensemble of randomly-distributed clumps will also cancel out any nonspherical Kerr-type behavior.) Since a displacement of magnitude $|d\vec{r}|$ will have a radial component (with respect to any particular mass concentration) of $|d\vec{r}|\cos\theta$, with a randomly-distributed value for the angle, the angle-averaging of the perturbation term therefore requires one to average over $dr^2 = |d\vec{r}|^2 \cos^2\theta = (dx^2 + dy^2 + dz^2)\cos^2\theta$. Since $\langle\cos^2\theta\rangle = (1/3)$ in three spatial dimensions, this means that the summed, averaged value of the spatial part of the overall metric perturbation due to a large number of randomly-located clumps will be multiplied by the factor $(1/3)$; though the temporal part (i.e., the perturbation to g_{tt}) will remain unaffected by such angle-averaging. This is due to the fact that only the radial projection of a given translation (with respect to a particular mass concentration) will 'feel' the perturbation potential from that clump in its contribution to the interval ds^2 (cf. Equation 7), while the full strength of the gravitational potential always contributes to the temporal part of the metric, regardless of directional configuration.

Consider now the observer to be located at the origin, surrounded by a set of (roughly identical) discrete mass concentrations with *total* mass M, all located within a particular spherical shell at coordinate distance r' from the origin, but randomly distributed in (θ, ϕ). From the above arguments, we can write the total, *linearly* summed and angle-averaged metric for this observer in 'isotropized' fashion, as[7]:

$$ds^2 = -c^2\{1 - [R_{\mathrm{Sch}}(t)/r']\}\, dt^2 \; + \; [a_{\mathrm{MD}}(t)]^2\{1 + (1/3)[R_{\mathrm{Sch}}(t)/r']\}\, |d\vec{r}|^2 \;, \qquad (8)$$

where $R_{\mathrm{Sch}}(t) \equiv \{(2GM/c^2)/[a_{\mathrm{MD}}(t)]\}$, and $|d\vec{r}|^2 \equiv (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) = |d\vec{x}|^2 \equiv (dx^2 + dy^2 + dz^2)$.

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[7]Note that this result for the overall summed perturbation is mathematically no different from what one would get by integrating a smooth spherical shell of continuous matter; but by thinking of mass M as being an assortment of many individual, vorticity-stabilized discrete sources, it becomes clearer to understand how the same actual matter – considering its mass *plus* its dynamical effects – can be responsible both for the background matter-dominated FRW metric, *and* for the perturbative metric terms superposed on top of it.

Looking at this expression, it seems physically reasonable to regard the term multiplying $|d\vec{r}|^2$ as a 'true' increase in spatial volume, when comparing the volume of a spatial hypersurface at two different cosmic times; and to regard the g_{tt} term as an 'observational' term, slowing down the perceptions of observers – relative to the expansion of the universe, governed by cosmic time t – at all times after inhomogeneities have begun to develop (including at the current time, $t = t_0$). Significantly, even if the spatial term by itself may not be enough to generate a real volumetric acceleration, once it is coupled with the temporal term the entire perturbation may indeed be enough to create a so-called "apparent acceleration" that is sufficient to explain all of the relevant cosmological observations.

The factor of $(1/3)$ in the perturbative term for the spatial metric components (relative to that for g_{tt}) may seem intriguing because it resembles the initial claim (later retracted) by Bean (2009) of $(1/\eta) \equiv (\psi/\phi) \equiv (\Phi_{\mathrm{Time}}/\Phi_{\mathrm{Space}}) \simeq 3$ from their analysis of the weak lensing shear field of the Hubble Space Telescope COSMOS data (Massey et al. 2007), appearing (briefly) to indicate a deviation from general relativity, which predicts $\psi = \phi$ in the absence of anisotropic stress. But this factor of $(1/3)$ in our Equation 8 is not 'fundamental', but merely the result of approximating the linearized sum of many individual 'Newtonian' solutions; this effectively spreads the total spatial perturbation among all three spatial metric terms, rather than confining it solely to g_{rr}, as is usual when considering a single inhomogeneity. Thus the general relativistic expectation of equal temporal and spatial potentials – i.e., $\psi = \phi$ – is not really violated here, and no actual new physics or modified gravity is implied by it.

Implications aside, Equation 8 just represents the perturbations to the metric due to masses at some specific coordinate distance r' – and thus from a specific look-back time t' – as seen from some particular observational point. The total metric at that spacetime point must be computed via an integration over all possible distances, out to the distance (and thus look-back time) at which the universe had been essentially unclustered. Finally, a light ray reaching us from its source (e.g., a Type Ia supernova being used as a standard candle) travels to us in a path composed of a collection of such points, where the metric at each point must be calculated via its own integration out to its individual "inhomogeneity horizon"; and only by calculating the metric at every point in the pathway from the supernova to our final location here at $r = z = 0$ can we figure out the total distance that the light ray has been able to travel through the increasingly perturbed metric, given its emission at some specific redshift z.

Consider a light ray emitted by a supernova at cosmic coordinate time $t = t_{\mathrm{SN}}$, which then propagates from the supernova at $r = r_{\mathrm{SN}}$, to us at $r = 0$, $t = t_0$. We refer here to the geometry depicted in Figure 2.

For each point $P \equiv (r, t)$ of the trajectory, the metric at that point will be perturbed

away from the background FRW form by all of the virialized clumps that have entered its causal horizon by that time. Consider a sphere of (coordinate) radius α, centered around point P, with coordinates $(\alpha, t_{\rm ret})$ (where $t_{\rm ret} \leq t$ is the retarded time), defined such that the information about the state of the clumping of matter on that sphere at time $t_{\rm ret}$ will arrive – via causal updating, traveling at the speed of null rays – to point P at the precise time t. To compute the fully-perturbed metric at P, we must integrate over the clumping effects of all such radii α, from $\alpha = 0$ out to $\alpha_{\rm max}$, the farthest distance from P out from which clumping information can have causally arrived since the clustering of matter had originally begun in cosmic history.

To really determine the relationships between $(\alpha, t_{\rm ret})$, $\alpha_{\rm max}$, and (r, t) with precision, we would need to compute the propagation of null rays from such $(\alpha, t_{\rm ret})$ to $(\alpha = 0, t)$ recursively or iteratively, since the propagation time of a null ray carrying perturbation information would itself be affected by all of the other perturbation information coming in to cross its path from everywhere else, during all times prior to arrival. In other words, causal updating is itself slowed by the metric perturbation information carried by causal updating, creating an operationally nonlinear problem. But out of necessity for this initial, proof-of-principle paper, we avoid this "recursive" (as opposed to gravitational) nonlinearity through the convenience of assuming an *unperturbed*, flat, matter-dominated FRW cosmology for computing all causal updating effects for the metric at (r, t). This crucial simplification – in addition to the simplification of ignoring gravitational nonlinearities as well, since we compute the total metric perturbation terms via a trivial summation of individual contributions – is necessary for the purposes of this paper, though a more complete analysis (considering both nonlinearities) must eventually be employed in order to meet the standards of Precision Cosmology. For now, however, what we can do is to check at the end of calculations that these nonlinearities have not grown too severe as $z \to 0$ (they should not be for most of our simulation runs, below), and to note that the output cosmological parameters and fits from our calculations here will unavoidably possess some systematic theoretical uncertainty as a result of these approximations.

Now, for a FRW metric with $a(t) = a_0(t/t_0)^{2/3}$, the coordinate distance traveled by a null ray in the cosmic time span from t_1 to t_2 will be $\alpha \equiv (c/a_0) \int_{t_1}^{t_2} (t/t_0)^{-2/3} dt = [(3c/a_0)(t_0)^{2/3}(t_2^{1/3} - t_1^{1/3})]$. Defining $a_0 \equiv 3ct_0 = 2c/H_0$, and with $t_2 \equiv t$, $t_1 \equiv t_{\rm ret}$, we thus have: $\alpha = [(t/t_0)^{1/3} - (t_{\rm ret}/t_0)^{1/3}]$. We then turn this into a prescription for computing $t_{\rm ret}$ as a function of t and α (and implicitly of the present time, t_0), as follows:

$$t_{\rm ret}(t, \alpha) = t_0[(t/t_0)^{1/3} - \alpha]^3 \ . \tag{9}$$

Similarly, we can determine α_{max}, given some initial time t_{init} at which structure formation can be reasonably said to have started (i.e., $\Psi(t \leq t_{\mathrm{init}}) \equiv 0$):

$$\alpha_{\mathrm{max}}(t, t_{\mathrm{init}}) = [(t/t_0)^{1/3} - (t_{\mathrm{init}}/t_0)^{1/3}] \,. \tag{10}$$

Now, how the metric is affected at P by a spherical shell of material at coordinate radius α depends upon the state of clumping there at the appropriate retarded time: $\Psi[t_{\mathrm{ret}}(t, \alpha)]$. The total effect is computed by integrating all shells from $\alpha = 0$ out to $\alpha = \alpha_{\mathrm{max}}(t, t_{\mathrm{init}})$; but in order to compute the metric perturbation from each shell quantitatively, it is first necessary to relate this clumping function to an actual physical density of material.

As discussed above, we define the $\Psi(t)$ function as representing the dimensionless ratio of matter which can appropriately be defined as 'clumped' at a given time, expressed as a fraction of the total physical density. With the convenience of assuming a flat FRW cosmology as the initially unperturbed state, the total physical density at all times will merely be an evolved version of the unperturbed FRW critical closure density from early (pre-perturbation) times.

Recalling Equation 8, we have the perturbation term $[R_{\mathrm{Sch}}(t)/r'] = \{(2GM/c^2)/[r' \, a_{\mathrm{MD}}(t)]\}$, with $a_{\mathrm{MD}}(t) = a_0(t/t_0)^{2/3} \equiv c[18t^2/H_0]^{1/3}$. The value of M to use here is given by the clumped matter density at coordinate distance α, times the infinitesimal volume element of the shell. The clumped matter density at time t, as implied above, will equal $[\Psi(t)\rho_{\mathrm{crit}}(t)]$; and the volume element in the integrand for that shell is given by: $4\pi R_{\mathrm{phys}}^2 dR_{\mathrm{phys}} = 4\pi[a_{\mathrm{MD}}(t) \, \alpha]^2[a_{\mathrm{MD}}(t) \, d\alpha]$. (Note that any density-dilution effects in $[\Psi(t)\rho_{\mathrm{crit}}(t)]$ due to volume creation by virializing inhomogeneities will be precisely canceled by the corresponding volume increase of that spherical shell, leaving its (backreaction-effective) differential mass element, "dM", unchanged. On the other hand, there *should* be an extra distance factor multiplying the denominator of $[R_{\mathrm{Sch}}(t)/r']$, due to $\Phi(t)$, $R_{\mathrm{Sch}}(t) \neq 0$; but we must drop it here in this simplified treatment in which we neglect "recursive nonlinearities".)

Collecting these terms (and letting $t' \equiv t_{\mathrm{ret}}(t, \alpha)$), the integrand will thus be equal to:

$$\begin{aligned}
[R_{\mathrm{Sch}}(t)/r']_{r'=\alpha \to (\alpha+d\alpha)} &= \{(2G/c^2) \, dM \, / \, [a_{\mathrm{MD}}(t) \, \alpha]\} & \text{(11a)} \\
&= \{(2G/c^2) \, [a_{\mathrm{MD}}(t) \, \alpha]^{-1} \, [\Psi(t') \, \rho_{\mathrm{crit}}(t)] \, [4\pi R_{\mathrm{phys}}^2 dR_{\mathrm{phys}}]\} & \text{(11b)} \\
&= \{(8\pi G/c^2) \, \Psi(t') \, [a_{\mathrm{MD}}(t) \, \alpha]^{-1} \, [\rho_{\mathrm{crit}}(t) \, [a_{\mathrm{MD}}(t)]^3] \, [\alpha^2 d\alpha]\} & \text{(11c)} \\
&= \{(8\pi G/c^2) \, \Psi(t') \, a_{\mathrm{MD}}(t)^{-1} \, [\rho_{\mathrm{crit}}(t_0) \, a_0^3] \, [\alpha d\alpha]\} & \text{(11d)} \\
&= \{(8\pi G/c^2) \, \Psi(t') \, [(t_0/t)^{2/3} \, (3ct_0)^{-1}] \, \{[3H_0^2/(8\pi G)] \, (3ct_0)^3\} \, [\alpha d\alpha]\} & \text{(11e)} \\
&= \{12 \, \Psi(t') \, [(t_0/t)^{2/3}] \, [\alpha d\alpha]\} \,, & \text{(11f)}
\end{aligned}$$

where for simplification we have used $H_0 = (2/3)t_0^{-1}$ and the fact that $[\rho(t)a(t)^3]$ is constant (or effectively so, as explained above), both true for a matter-dominated universe. Note also that only Ψ is evaluated at the retarded time, $t_{\mathrm{ret}}(t, \alpha)$. The *strength* of the metric perturbation (at time t for point P) for a point-like Newtonian perturbation embedded in the expansion actually depends upon its instantaneous physical distance from P at t, as is obvious from $\Phi(t) = \{-GM/[a_{\mathrm{MD}}(t)\ r]\} \simeq [-GM/R_{\mathrm{phys}}(t)]$ in Equation 7. The only "relativistic" piece of propagating information which is causally delayed is the state of clumping, $\Psi[t_{\mathrm{ret}}(t, \alpha)]$, that has just then arrived from coordinate distance α to observer P at (r, t).

From this result, we can now determine the total integrated metric perturbation function due to clumping, $I(t)$, as experienced by a null ray (or any observer) passing through point P at (r, t):

$$I(t) = \int_0^{\alpha_{\mathrm{max}}(t, t_{\mathrm{init}})} \{12\ \Psi[t_{\mathrm{ret}}(t, \alpha)]\ [(t_0/t)^{2/3}]\}\ \alpha\ d\alpha\ , \tag{12}$$

with $I(t)$ implicitly being a function of t_{init} (with $I(t) \equiv 0$ for $t \leq t_{\mathrm{init}}$), as well as of t_0.

Finally, we can insert this result back into the formalism of Equation 8, to obtain the final clumping-perturbed metric that we will use for all of our subsequent cosmological calculations:

$$ds^2 = -c^2[1 - I(t)]\ dt^2\ +\ \{[a_{\mathrm{MD}}(t)]^2\ [1 + (1/3)I(t)]\}\ |d\vec{r}|^2\ , \tag{13}$$

Representing a smoothly-inhomogeneous universe, as it does, this metric depends only upon time, and is thus equally good anywhere in the modeled universe – in particular, at every point in the trajectory of a light ray from a distant supernova to us.

A few important comments must be made about this result. First, note that the integrand for $I(t)$ in Equation 12 would actually become infinite as $\alpha \to \infty$, were it not limited by the finite causal horizon for seeing perturbations (i.e., finite α_{max}), and by the lessened degree of clumping as one looks back to earlier retarded times, deeper in the past (i.e., $\Psi(t \to t_{\mathrm{init}}) \simeq 0$). This implies that the integrated result for $I(t)$ can indeed become quite large at late times for Newtonian-level perturbations alone, given a large degree of clumping, in accord with our discussion from Section 2.3; and also that the dominant contribution to $I(t)$ will typically come from the largest coordinate distance out to which one can still see a substantial degree of clumping at its associated t_{ret}, indicating (as we will see) that clustering models $\Psi(t)$ with stronger clumping earlier on will have a much more powerful overall perturbative effect.

Second, we should reiterate the various approximations that have been made in order to obtain this smoothly-inhomogeneous cosmological metric – beyond, of course, the

core assumption of spatially-random clustering. These include the gravitationally-linearized treatment in the summing together of independent metric perturbations (only valid for $I(t)$ sufficiently less than unity, as will be examined later in Section 6.1), and the dropping of the recursive nonlinearities inherent to the physical process of causal updating. Such effects would eventually need to be included to produce a high-precision model of this type of back-reaction, from individually-Newtonian, virialized structures. But too-sophisticated a model along these lines does not necessarily make sense, since at some point the basic assumption of spatial randomness itself breaks down, requiring the abandonment of this smoothly-inhomogeneous formalism entirely in favor of a fully 3D cosmic structure simulation program – perhaps along the lines of Springel et al. (2005), for example, with Newtonian-level backreaction effects and causal updating added in. Since such a 3D simulation model is far beyond the scope of this paper, we will consider it sufficient here to stick with Equation 13 as a reasonable first-order approach to the problem, with all approximations and caveats kept in mind.

Lastly for this subsection, we note that the metric in Equation 13 has been made to look very much like an ordinary FLRW metric; and it can be made to look exactly like one with the transformation $dt' \equiv \sqrt{1 - I(t)} \, dt$, and with an appropriate redefinition of the scale factor, $a'(t') \equiv \{a_{\mathrm{MD}}[t(t')] \, \sqrt{1 + (1/3)I[t(t')]}\}$. One might then be tempted to conclude that observing an acceleration in $a'(t')$, just like in the usual FLRW case, still requires some form of Dark Energy violating the strong energy condition. But this would be incorrect, because the cosmological model that we have produced here is not simply a true FLRW metric shown in different guises via coordinate transformations, but in fact is a very physically different (dynamically inhomogeneous) model that is merely being approximated with FLRW-like averaged parameters. The detailed astrophysical effects being averaged into it, in contributing to the 'real' volumetric effects in g_{rr} and/or to the 'observational' signal-delaying effects in g_{tt}, are themselves capable of combining together sufficiently to create an apparent acceleration in $a'(t')$ – even without any SEC-violating component, or any actual accelerating patch of spacetime as might be measured by observers in a local reference frame.

3.3. Light Propagation, Redshifts, and Luminosity Distances with Causal Updating

Due to the altered mathematical form of Equation 13 with respect to the pure FLRW case, the observed cosmological evolution and parameters will differ from what would normally be expected given some usual $a(t)$ function. One must distinguish between parameters which merely refer to the underlying theoretical FRW (unperturbed, no Dark Energy)

model that holds for the pre-clumping universe – that is, the "bare" parameters – versus those ("dressed") parameters which are obtained from current-day observations. We will use the superscript "FRW" for the former, and the superscript "Obs" for the latter, and will compute the relationships between these different parameter sets for observables like redshift, the Hubble Constant, the Age of the Universe, and other important variables.

We begin by focusing upon parameters used for computing the supernova-based luminosity distance function that most directly traces out the cosmic expansion history. First, one must calculate how observed redshifts are altered in this model. Looking at Equation 13, which is in the form $ds^2 = -g_{tt}(t)\, dt^2 + g_{rr}(t)\, |d\vec{r}|^2$, we note again that one could transform away the $g_{tt}(t)$ function with a redefinition of the time coordinate, and thus redshifts can be calculated here simply by taking $\sqrt{g_{rr}(t)}$ as the new scale factor. In other words, given the standard relationship for the "FRW" variables:

$$z^{\mathrm{FRW}}(t) \equiv \frac{a_{\mathrm{MD}}(t_0)}{a_{\mathrm{MD}}(t)} - 1 = (t_0/t)^{2/3} - 1 \; , \tag{14}$$

we can similarly write:

$$z^{\mathrm{Obs}}(t) \equiv \frac{\sqrt{g_{rr}(t_0)}}{\sqrt{g_{rr}(t)}} - 1 = [\sqrt{\frac{1 + (1/3)I(t_0)}{1 + (1/3)I(t)}}\, (t_0/t)^{2/3}] - 1 \; , \tag{15}$$

where we define $t_0 \equiv t_0^{\mathrm{FRW}}$, $t_{\mathrm{init}} \equiv t_{\mathrm{init}}^{\mathrm{FRW}}$; and we will mean $t_x \equiv t_x^{\mathrm{FRW}}$, $z_x \equiv z_x^{\mathrm{FRW}}$, for time coordinate and redshift values in general, except when expressly referring to them in the form t_x^{Obs} or z_x^{Obs}.

Now, in order to calculate observed luminosity distances, we must compute the coordinate distance r of a supernova going off at coordinate time t, for which a light ray would be arriving here (at $r = 0$) precisely at t_0. The modification of $r(t)$ imposed here as a perturbation to the FRW result, essentially due to the summed Shapiro time delays (e.g., Weinberg 1972) contributed by all causally-seen virialized clumps, is the major effect of inhomogeneities that we consider in this paper, since we neglect other effects like lensing along beam paths (e.g., Kantowski 2003), and so on, which are likely too small to generate an observed cosmic acceleration. For a null ray, with $ds^2 = 0$, and considering pure inward radial motion ($|d\vec{r}|^2 \to dr^2$, $dr/dt < 0$), we have $dr/dt = -\sqrt{g_{tt}(t)/g_{rr}(t)}$. We can thus compute the coordinate distance of the supernova from us, as a function of t, as follows (with $I(t)$ still as computed via Equation 12):

$$r_{\mathrm{SN}}^{\mathrm{FRW}}(t) \;\; \equiv \;\; |r^{\mathrm{FRW}}(t_0) - r^{\mathrm{FRW}}(t)| = \int_t^{t_0} \{\sqrt{\frac{g_{tt}(t')}{g_{rr}(t')}}\}\, dt' \tag{16a}$$

$$= \int_t^{t_0} \left\{ \frac{c}{a_{\mathrm{MD}}(t')} \sqrt{\frac{1 - I(t')}{1 + (1/3)I(t')}} \right\} dt' \tag{16b}$$

$$= \frac{c}{a_0} \int_t^{t_0} \left\{ (t_0/t')^{2/3} \sqrt{\frac{1 - I(t')}{1 + (1/3)I(t')}} \right\} dt' \ . \tag{16c}$$

This coordinate distance function can then be converted into an expression for the observed luminosity distance. For the perfectly homogeneous FRW case, the luminosity distance of a standard candle is given by the current physical distance to it, times a redshift factor: $d_{\mathrm{L,FRW}} = [a_0 \, r_{\mathrm{SN}} \, (1 + z)]$. For our inhomogeneity-perturbed model, all of the appropriate time dilation/redshift factors are taken care of by using $z^{\mathrm{Obs}}(t)$ from Equation 15; and the physical distance can be given by $r_{\mathrm{SN}}^{\mathrm{FRW}}(t)$ and the modified scale factor, as follows:

$$d_{\mathrm{L,Pert}}(t) = \left[a_0 \sqrt{1 + (1/3)I(t_0)} \right] r_{\mathrm{SN}}^{\mathrm{FRW}}(t) \, [1 + z^{\mathrm{Obs}}(t)] \tag{17a}$$

$$= \frac{1 + (I_0/3)}{\sqrt{1 + [I(t)/3]}} \frac{c \, t_0^{4/3}}{t^{2/3}} \int_t^{t_0} \left\{ (t')^{-2/3} \sqrt{\frac{1 - I(t')}{1 + [I(t')/3]}} \right\} dt' \tag{17b}$$

$$= \frac{1 + (I_0/3)}{\sqrt{1 + [I(t_r)/3]}} \frac{c \, t_0}{t_r^{2/3}} \int_{t_r}^{1} \left\{ (t_r')^{-2/3} \sqrt{\frac{1 - I(t_r')}{1 + [I(t_r')/3]}} \right\} dt_r' \ , \tag{17c}$$

where $I_0 \equiv I(t_0)$, and t_r, t_r' are dimensionless time ratios (e.g., $t_r \equiv t/t_0$), with no change to the essential form of $I(t)$ (i.e., $I(t) = I(t_r \cdot t_0) \Rightarrow I(t_r)$). Note that this $d_{\mathrm{L,Pert}}(t)$ is simply proportional to $(ct_0) = [(2/3) \, c/H_0^{\mathrm{FRW}}]$, as would be expected.

With this expression for $d_{\mathrm{L,Pert}}(t)$, and Equation 15 for $z^{\mathrm{Obs}}(t)$, one could in theory combine them analytically to produce $d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})$, the function actually observed in supernova luminosity distance (i.e., Hubble) curves. But this is neither analytically nor computationally practical, and so we have instead performed our numerical calculations using arrays for many discrete points in t, utilizing the above formulae to evaluate arrays for $d_{\mathrm{L,Pert}}(t)$ and $z^{\mathrm{Obs}}(t)$, which we then simply combine together into one array as $d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})$. It should therefore be noted that all of our subsequent plots of luminosity distance curves in this paper for different clustering evolution models, though presented as smooth curves, are in fact connected dots of points evaluated for discrete values of t^{FRW}.

Our pixelization, which we have rigorously tested to ensure accurate results, typically employs ∼2150 discrete time values, most of which (∼1500) are concentrated in the more recent times ($z^{\mathrm{FRW}} \lesssim 1$) where most of the dynamical evolution is happening; fewer pixels are reserved for earlier times, going back to before clumping had started when the simple FRW model was still correct. Increasing the number of discrete time values used, even by

a factor of 5, results in only a small change (e.g., $\lesssim 0.15\%$) for all cosmological fits and parameters, even those requiring up to three derivatives of the luminosity distance function.

Given that this discrete version of $d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})$ must in fact be differentiated to obtain cosmological parameters from it (see Section 5 below), we do so by using the definition of the derivative for each pixel. That is, to get the ith pixel entry (i.e., evaluated at $t_{\{i\}}$) for the Nth derivative of $d_{\mathrm{L,Pert}}$ with respect to z^{Obs}, we compute it as follows:

$$[d_{\mathrm{L,Pert}}^{N\prime}]_{\{i\}} = \frac{d}{d\,z^{\mathrm{Obs}}}[d_{\mathrm{L,Pert}}^{(N-1)\prime}]_{\{i\}} \equiv \frac{d_{\mathrm{L,Pert}}^{(N-1)\prime}(t_{\{i+1\}}) - d_{\mathrm{L,Pert}}^{(N-1)\prime}(t_{\{i\}})}{z^{\mathrm{Obs}}(t_{\{i+1\}}) - z^{\mathrm{Obs}}(t_{\{i\}})} \ . \tag{18}$$

Note that each subsequent derivative array has one less pixel than the prior derivative array, since we lack an extra pixel at the (high-z) end to compute the last pixel of the differentiated array (e.g., if $d_{\mathrm{L,Pert}}'$ has 2149 pixels, then $d_{\mathrm{L,Pert}}''$ has 2148 pixels, and so on).

The calculation of currently observable cosmological parameters requires derivatives which are evaluated at $z = 0$, $t = t_0$. We simply use the first (lowest-z, highest-t) pixel for that derivative value:

$$[d_{\mathrm{L,Pert}}^{N\prime}]_{(z\to0,\ t\to t_0)} \equiv [d_{\mathrm{L,Pert}}^{N\prime}]_{\{1\}} \ . \tag{19}$$

This appears to be a robust procedure, since the arrays of derivative values are well-behaved everywhere (except for transient jumps at transitions of our piecewise-continuous input $\Psi(t)$ functions at high-z), and are smooth heading towards $t \to t_0$. Moreover, many simulation runs with a variety of different clumping function time-dependencies and amplitudes have given results for these cosmological parameters which seem to be reasonable and mutually consistent.

Ultimately, luminosity distance functions for standard candles are plotted as residual Hubble diagrams, where one logarithmically plots the ratio of d_{L} to the luminosity distance function for a coasting universe of the same current expansion rate. More precisely, we are interested in the function $\Delta(m - M) = 5\{\mathrm{Log}_{10}[d_{\mathrm{L,Data}}(z)] - \mathrm{Log}_{10}[d_{\mathrm{L,Coast}}(z)]\}$, where $d_{\mathrm{L,Coast}}(z) = [(c/H_0)(1 + z)\mathrm{Ln}(1 + z)]$. But, *which* H_0 does one use here for subtracting off the coasting universe from our numerically simulated perturbed-universe models? Although we see from Equation 17 that $d_{\mathrm{L,Pert}}$ is indeed proportional to $t_0 \propto (1/H_0^{\mathrm{FRW}})$, that is not sufficient to tell us the real observed expansion rate that is asymptotically approached as $z \to 0$ (i.e., $H_0^{\mathrm{Obs}} \neq H^{\mathrm{FRW}}$), since the integral expression could evaluate to practically anything, given the appropriate clumping evolution function.

To determine the proper value to use for the $z \to 0$ expansion rate, consider Equation 2.51 of Kolb & Turner (1990), from which one obtains the general approximation $d_{\mathrm{L}} \simeq cz/H_0$ for very small z. This gives an operational definition for the observed expansion rate, H_0^{Obs},

via:

$$d'_{\mathrm{L,Pert,0}} \equiv \{d[d_{\mathrm{L,Pert}}]/d[z^{\mathrm{Obs}}]\}_{z \to 0} \equiv c/H_0^{\mathrm{Obs}} \ . \tag{20}$$

(We will later use this general method to explicitly relate H_0^{Obs} to H_0^{FRW}, and to find similar relationships for other cosmological observables, in Section 5.) Using this prescription, we can finally define the residual Hubble diagram function for our clumping-perturbed model, as follows:

$$\Delta(m-M)_{\mathrm{Pert}}(z^{\mathrm{Obs}}) = 5 \ \{ \ \mathrm{Log}_{10}[d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})] \ - \ \mathrm{Log}_{10}[(d'_{\mathrm{L,Pert,0}})(1+z^{\mathrm{Obs}})\mathrm{Ln}(1+z^{\mathrm{Obs}})] \ \} \ . \tag{21}$$

A straightforward series expansion demonstrates that we have correctly specified the formula for the observed expansion rate, H_0^{Obs}, in order to match the expansion velocities of the coasting and clumping-perturbed models at t_0.

Using Equation 21 and the rest of the expressions in this section, we are now able to convert any clumping evolution function $\Psi(t^{\mathrm{FRW}})$ into a residual Hubble diagram that can be compared with any theoretical FLRW model that one chooses (e.g., ΛCDM models), as well as with real standard candle data from Type Ia supernova observations.

4. MODELS, PARAMETERS, AND RESULTS

4.1. Selection of Clumping Evolution Functions

In order to produce cosmological predictions with our formalism, one must first choose a set of $\Psi(t^{\mathrm{FRW}})$ functions to evaluate which are reasonable models of how the fraction of cosmic matter in the clumped state has evolved over time. Choosing likely functions is not as trivial as it may seem, however. For example, while the contrast of a density variation will evolve as $\delta\rho/\rho \propto a(t) \propto t^{2/3}$ in the linear regime for a matter-dominated universe (e.g., Kolb & Turner 1990), this only represents the linear evolution of a single clump; it says nothing about the initial development of new clumps (often due to collisions), or about the nonlinear regime and virialization for very dense clumps. Our clumping evolution functions, however, must serve as proxies for emulating all of these effects combined together.

As one possibility, one could perhaps look at clumping evolution functions that are output from large cosmological structure formation simulations (e.g., Springel et al. 2005). But since such models basically use the instantaneous Poisson Equation formalism (recall Equations 4-6), without causal updating, they lack the realism necessary to properly simulate the evolution of clustering without the use of a Dark Energy "fudge factor", which complicates the interpretation of such results.

This author's first approach in defining $\Psi(t^{\mathrm{FRW}})$ was to consider observational data directly, for guidance, treating the star formation rate (SFR) as a tracer of the rate of the increase in clumping – i.e., $d[\Psi(z)]/dz \propto SFR(z)$. This method of choosing likely $\Psi(t)$ functions was indeed capable of finding some which reproduced the supernova Hubble curves with reasonable success (results not shown here); but given the large uncertainties in the observationally measured SFR power-law parameters (e.g., Glazebrook et al. 2003), and the large number of arbitrarily-tunable parameters in the $\Psi(t)$ functions adapted from such data, the actual statistical significance of a 'good fit' was too difficult to meaningfully determine.

Given these difficulties, we decide here to opt for simplicity in choosing which clumping evolution models to use for the main runs of our numerical simulation program in this analysis. Obviously, there is nothing simpler to use than $\Psi(t) \propto t$, a fairly sensible choice to begin with, since one would assume that the amount of clumping that can occur should to some degree depend directly upon how much time is available for that clumping to develop. For some alternatives, we have also chosen to use models with $\Psi(t) \propto t^{2/3}$, as this is proportional to the linear density contrast evolution in a matter-dominated universe; as well as models with an 'accelerating' clumping rate, $\Psi(t) \propto t^2$, to test whether that would possibly help in creating an observed acceleration. This latter time-dependency also potentially corresponds to the final *nonlinear* evolution of a density perturbation (Kolb & Turner 1990, p. 322).

Quantitatively, we define our three different classes of clumping evolution models as follows:

$$
\Psi_{\mathrm{Lin}}(t) \;\; \equiv \;\; \Psi_0 \, (\frac{t - t_{\mathrm{init}}}{t_0 - t_{\mathrm{init}}}) \tag{22a}
$$

$$
\Psi_{\mathrm{MD}}(t) \;\; \equiv \;\; \Psi_0 \, (\frac{t - t_{\mathrm{init}}}{t_0 - t_{\mathrm{init}}})^{2/3} \tag{22b}
$$

$$
\Psi_{\mathrm{Sqr}}(t) \;\; \equiv \;\; \Psi_0 \, (\frac{t - t_{\mathrm{init}}}{t_0 - t_{\mathrm{init}}})^2 \; , \tag{22c}
$$

where t_{init} represents the beginning of clumping, such that $\Psi(t \leq t_{\mathrm{init}}) \equiv 0$ for all models; and $\Psi_0 \equiv \Psi(t_0)$ represents the current state of clumping today. Note that all of these functions are defined in terms of t^{FRW} (and hence z^{FRW}), rather than in terms of t^{Obs} or z^{Obs}; the latter would of course be preferable, though it is not possible here because t^{Obs} and z^{Obs} depend recursively upon $\Psi(t)$.

For these three different classes of models, we have two physically meaningful parameters to vary: Ψ_0 and t_{init} (though we will usually express the beginning of clumping in terms of z_{init}, which can be obtained from t_{init} via Equation 14). Now, while this gives us a fairly wide region of parameter space to explore through in trying to match the observed supernova data, results that succeed in matching the data are still meaningful, since these model input parameters are constrained by astrophysical considerations; and also because (as will be seen

below) these classes of models have characteristic behaviors that quite naturally look very much like ΛCDM cosmologies over a wide range of redshifts and model parameter choices.

4.2.   Numerical Results: Residual Hubble Diagrams and Supernova Data Fits

In designing a suite of simulation runs to test the effectiveness of our model at reproducing the observed cosmic acceleration, we use established observational data as our guide for specifying interesting choices for parameters z_{init} and Ψ_0.

With z_{init} representing the beginning of the cosmic phase transition from smooth to clumped, it seems reasonable to associate z_{init} with another symptom of the beginning of structure formation: the epoch during (or slightly before) the onset of cosmological reionization.

The five-year WMAP Data analysis (Dunkley et al. 2009) supported a value of $z_{\mathrm{reion}} \simeq 11$ in the case of an instantaneous reionization; but the data also suggests the possibility of an extended period of partial reionization, perhaps beginning as early as $z \sim 20$, and extending no later than $z \sim 6$. To broadly cover this range (and to bracket it, to be conservative), we have chosen values of $z_{\mathrm{init}} = (5, 10, 15, 25)$ for our simulation runs, with the larger values of z_{init} generally being more astrophysically interesting as starting times for the transition to clumping.

Specifying appropriate values of Ψ_0 is a more subtle task, though, since one has to quantitatively characterize the much more complex dynamical situation of late-time clustering with a single number. Furthermore, we must rely a great deal upon our assumption of a smoothly-inhomogeneous universe, since the existence of a local void or bubble would seriously alter the limiting behavior of $\Psi(z \to 0)$. Nevertheless, we are able produce a range of parameter values which make general astrophysical sense, and also turn out to produce good cosmological results.

First, consider that while observations may tell us that, say, $\Omega_{\mathrm{M}}^{\mathrm{Obs}} \equiv 1 - \Omega_{\Lambda}^{\mathrm{Obs}} \sim 0.27$ with $\Omega_b^{\mathrm{Obs}} \sim 0.04$ (and thus $\Omega_{\mathrm{DM}}^{\mathrm{Obs}} \sim 0.23$), our model here is one of a flat, apparently accelerating universe, with *no* dark energy. Thus we exploit the fact (details to be given in Section 5.2 below) that $H_0^{\mathrm{Obs}} \neq H_0^{\mathrm{FRW}}$, in order to achieve $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \equiv 1$ without ever changing the actual physical matter density ($\omega_{\mathrm{M}} \propto \rho_{\mathrm{M}}$) that can be more directly determined through other observations (growth of structure, cluster mass measurements, etc.). Furthermore, since the physical baryon density ω_b, and its relationship to ω_{DM}, are fairly well tested by the CMB peak height ratios (Bennett et al. 2003) regardless of whatever model is used to produce acceleration at later times, it is therefore wise not to change this ratio, $\omega_b/\omega_{\mathrm{DM}}$. A

simple scaling up to flatness ($\Omega_{\mathrm{Tot}}^{\mathrm{FRW}} \equiv 1$) thus gives us $\Omega_b^{\mathrm{FRW}} \sim [0.04(1.0/0.27)] \simeq 0.15$, and $\Omega_{\mathrm{DM}}^{\mathrm{FRW}} \simeq 1 - \Omega_b^{\mathrm{FRW}} \simeq 0.85$.

We must then determine reasonable estimates for how much of each species, Dark and baryonic matter, might be clumped at the present time, noting (as per Section 3.2) that Ψ_0 is given simply as a dimensionless fraction of the total matter density.

At one extreme, we may treat the universe as 'completely clumped' at the present time – i.e., $\Psi_0 = 1$.

At the other extreme, we may consider almost all baryonic matter to still be unclumped, due to 'gastrophysics' like shock heating and its resultant thermal pressure; exceptions being only that amount of baryonic matter clumped into stars, and perhaps (depending upon how to appropriately define 'clumped') the amount of gas bound into virialized galaxies. Dark matter, on the other hand, not being subject to ordinary thermal pressure (and being able to virialize gravitationally), would be almost entirely clumped (for Dark Matter clustering, see, e.g., Gilmore et al. (2007)), *except* for some small portion of it that would actually be Hot Dark Matter (neutrinos).

Getting precise numbers for these quantities is not trivial, but estimates are available: for example, Turner (2002b) estimates $\sim$1/8 of the baryonic matter to be contained in stars, so that (for our model) $\Omega_{\mathrm{stars}} \sim 0.0185$. Also, Hinshaw et al. (2009) limits the neutrino physical density to being less than either $\sim$12 − 13% of the total dark matter density (WMAP data only), or $\sim$5−6% of it (WMAP+BAO+SN combined data), which corresponds (respectively) in our model to $\Omega_\nu \lesssim 0.1$ or $\Omega_\nu \lesssim 0.05$.

Thus, summarizing the total density budget, one ends up with $\sim$2% locked up into stars, $\sim$13% remained as either clumped or unclumped baryonic gas, $\lesssim$ 10% (or $\lesssim$ 5%) as unclumped neutrinos, and $\gtrsim$ 75% (or $\gtrsim$ 80%) as mostly clumped Cold Dark Matter.

We can therefore classify $\sim$77 − 82% of the mass as 'probably clumped', $\lesssim 5 - 10\%$ as 'probably unclumped', with $\sim$13% or so as some mix of both. With this information, and to space out our parameters fairly evenly, we have chosen values of $\Psi_0 = (0.78, 0.85, 0.92, 0.96, 1.0)$ for our simulation runs, with the mid-range values of Ψ_0 likely being the most astrophysically sensible.

With four different values of z_{init}, five different values of Ψ_0, and three different clumping evolution models, this gives us $4 \times 5 \times 3 = 60$ simulation runs in total. Residual Hubble diagrams have been computed for all of these runs, and are plotted below, with Figure 3 showing the results for the Ψ_{Lin} runs, Figure 4 depicting the Ψ_{Sqr} runs, and Figure 5 depicting the Ψ_{MD} runs. Each figure includes four panels, with the panels representing $z_{\mathrm{init}} = (5, 10, 15, 25)$ in

order from top to bottom. Within each panel itself, the FLRW cases of the flat SCDM model ($\Omega_M = 1$) and a Concordance ΛCDM model ($\Omega_\Lambda = 0.73 = 1 - \Omega_M$) are shown for comparison against five of our models with different degrees of clumping: $\Psi_0 = (0.78, 0.85, 0.92, 0.96, 1.0)$ in order from the lowest curve to highest.

A detailed quantitative analysis of the best of these runs in terms of quality of fit to real supernova data, and with several important cosmological parameters being computed for each run, will be presented below in Section 6. For now, though, we make the important qualitative observation that this formalism *works.* Not only does it yield a selection of several models possessing a perturbative effect strong enough to reproduce the observed (apparent) acceleration, but it produces curves that clearly do behave very much like ΛCDM – in particular, the Ψ_{Sqr} runs look like flat ΛCDM with $\Omega_\Lambda \sim 0.3 - 0.4$, the Ψ_{Lin} runs look like flat ΛCDM with $\Omega_\Lambda \sim 0.5 - 0.8$, and the Ψ_{MD} runs look like flat ΛCDM with $\Omega_\Lambda \sim 0.65 - 0.97$.

(Ironically, the 'accelerated' clumping models, Ψ_{Sqr}, produce the weakest observed acceleration effect, because they have less clumping at early times. Clumping is more important the earlier it occurs, because of a geometric effect: a longer look-back time for the beginning of clumping results in a much larger horizon out to which an observer can see substantial inhomogeneities; and it is the huge volume of this outer shell of perturbations that produces the strongest effect on the observer, as pointed out towards the end of Section 3.2.)

Most importantly, even without doing a search over our parameter space for 'best-fit' models – just by choosing a set of simple clumping evolution models and astrophysically-reasonable parameters for input into our model – we find that a significant number ($\sim$10 or so) of these 60 simulation runs manage to fairly precisely reproduce the Concordance ΛCDM Hubble curve. Furthermore, as will be shown shortly, these runs are able to fit the supernova data essentially as well as (and in certain cases, with some model parameter optimization, even better than) such best-fit flat ΛCDM Hubble curves that have heretofore been used to argue for the existence of Dark Energy.

5. FORGING A NEW CONCORDANCE FOR A SMOOTHLY-INHOMOGENEOUS UNIVERSE

The qualitative ability of our formalism to reproduce ΛCDM-like Hubble curves was shown in the previous section, and a more quantitative analysis demonstrating how these models are able to fit the observed supernova data will be given later on.

But beyond just succeeding at explaining the apparent acceleration seen in data like that from Type Ia supernovae (SNe), it is widely recognized that a true model of the universe must

satisfy the constraints imposed by several different, complementary cosmological data sets, while simultaneously producing a set of cosmological parameters that are consistent with all other relevant astronomical observations. Only such a fully consistent, astronomically-correct cosmological solution would attain the status of a 'concordance', sufficient to replace the well-known "Cosmic Concordance" representing the range of ΛCDM models that appear already to be cosmologically consistent, given the state of the data at this time.

In order to extract the appropriate cosmological parameters from our simulated Hubble curves, we must compute each of the relevant cosmological observables from $d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})$ as it was defined previously in Equation 17 (and subsequent discussion), with all derivatives (with respect to z^{Obs}) and limits of this (discrete) simulation output array performed as indicated via Equations 18-19. Brief derivations will be presented below of the expressions needed for converting model simulation results into observable parameters.

With such expressions in hand, we will be able to determine how these ("dressed") observables relate to unperturbed ("bare") model parameters such as t_0^{FRW}, H_0^{FRW}, and $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, for different choices of Ψ_0, z_{init}, and clumping evolution model Ψ_{Lin}, Ψ_{MD}, or Ψ_{Sqr}. These relationships are interesting not only because they give us hard numbers to use for comparison with real observations, but also because they reveal how much causal backreaction due to structure formation has altered the evolution of our universe from that predicted by purely homogeneous FRW models.

While it is beyond the scope of this paper to construct a completely new concordance in all of its aspects, in the subsections below we will show how to calculate several important cosmological parameters from our numerical simulations. Then, in Section 6, we will demonstrate the observed consistency of our best-fitting models with several of the key observational parameters of the ΛCDM Cosmic Concordance, without the use of any negative pressure species like Dark Energy.

5.1. H_0^{Obs}, Cosmic Proper Time, and the Age Problem

As discussed earlier in regards to Equations 20-21, an operational definition for the observed Hubble constant in terms of our simulation results can be given as $H_0^{\mathrm{Obs}} \equiv c/d'_{\mathrm{L,Pert,0}}$. For a normal, matter-dominated flat SCDM cosmology with no Dark Energy (and neglecting radiation), one has $H_0^{\mathrm{FRW}} \equiv [\dot{a}(t)/a(t)]_{t \to t_0} = (2/3)t_0^{-1}$. We can put this together to get the straightforward result:

$$H_0^{\mathrm{FRW}} = H_0^{\mathrm{Obs}} \left\{ \frac{2}{3} \frac{1}{ct_0^{\mathrm{FRW}}} \ d'_{\mathrm{L,Pert,0}} \right\} . \tag{23}$$

Now, we recall from Equation 17c that $d_{\mathrm{L,Pert}}(t)$ (and hence $d_{\mathrm{L,Pert}}(z^{\mathrm{Obs}})$, and all of its derivatives with respect to z^{Obs}) are simply proportional to ct_0^{FRW}; thus any dependence upon the parameter t_0^{FRW} cancels out (as it must) from Equation 23, and this formula merely gives us a dimensionless ratio between the observed and unperturbed Hubble Constants, H_0^{Obs} and H_0^{FRW}, the value of which depends upon the result of that numerical integration.

Note that it is H_0^{FRW} which we place on the left hand side of Equation 23, as the 'unknown' parameter; the value of H_0^{Obs} is set from real Hubble recession measurements (getting, for example, 72 km s^{-1}Mpc^{-1}), which one can then translate into H_0^{FRW} for any particular clumping evolution model, in order to obtain its 'true' cosmic expansion rate, as would have been observed if perturbations had never altered the observed expansion rate from its FRW value.

The importance of this relationship is that the expression in braces in Equation 23 will be less than unity (since $I(t) > 0$) when perturbations exist, thus resulting in $H_0^{\mathrm{FRW}} < H_0^{\mathrm{Obs}}$. This permits one to have a low value of H_0^{FRW} (say, in the 40's), while still retaining $H_0^{\mathrm{Obs}} \approx 72$. This possibility explains why several cosmological measurements may be quite concordant with a low Hubble constant, while removing the contradiction that such a result would seem to create for late-time measurements with standard candles (e.g., Freedman et al. 2001) that clearly indicate a high H_0^{Obs}. (For low-H_0 discussions, see for example Blanchard et al. (2003), Spergel et al. (2003), and Hunt & Sarkar (2007); and also see Figure 14 of Larson et al. (2011), showing the consistency of WMAP-only data with $\Omega_{\mathrm{M}} = 1$ models for H_0 in the $\sim$30's-40's.)

Having $H_0^{\mathrm{FRW}} \neq H_0^{\mathrm{Obs}}$ also makes other apparent conflicts go away, such as the classic Age Problem/Crisis in cosmology (e.g., Kolb & Turner 1990; Turner 2002b), in which a matter-dominated SCDM universe appears to be younger than some of its oldest constituents (e.g., globular clusters). The age of such a universe is $t_0 = (2/3)H_0^{-1}$, which for $H_0 \simeq$ 72 km s^{-1}Mpc^{-1} gives only $t_0 \simeq 9$ GYr, requiring one to assume an accelerating universe that had slower expansion in the past, in order to lengthen t_0 to $\sim$13 $-$ 14 GYr.

For our model, on the other hand, we can use the metric given above in Equation 13 to relate the 'observed' age of the universe, t_0^{Obs}, to t_0^{FRW}, as follows:

$$t_0^{\mathrm{Obs}} = \int_0^{t_0^{\mathrm{FRW}}} \{\sqrt{1 - I(t)}\} \, dt \ . \tag{24}$$

Now, even though this results in $t_0^{\mathrm{Obs}} < t_0^{\mathrm{FRW}}$, we also have $H_0^{\mathrm{FRW}} < H_0^{\mathrm{Obs}}$, so that the value of $t_0^{\mathrm{FRW}} \equiv (2/3)(1/H_0^{\mathrm{FRW}})$ will be much larger than that expected from FRW considerations (i.e., significantly larger than $\sim$13 $-$ 14 GYr); and thus when these two factors

are combined together, the result for t_0^{Obs} is able to fall precisely within the range necessary to solve the Age Problem, as we will show below in Section 6.1 for several of our best simulation runs.

5.2. Spatial Flatness and the Observed Matter Density: $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$ versus $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$

Another major argument used in favor of Dark Energy as part of the Cosmic Concordance, is the apparent contradiction between CMB peak data showing the universe to be spatially flat ($\Omega_{\mathrm{Tot}} \simeq 1$), while actual searches for the required amount of matter persistently turn up short on the overall density of clustering mass, looking instead like $\rho_{\mathrm{M}}/\rho_{\mathrm{crit}} \simeq 0.3$. The usual conclusion is that $\Omega_{\mathrm{M}} \simeq 0.3$, and that the gap of $(\Omega_{\mathrm{Tot}} - \Omega_{\mathrm{M}}) \simeq 0.7$ is filled by the existence of Dark Energy.

The unstated assumption in this reasoning, however, is that it is always accurate to use $\Omega_{\mathrm{M}} = \omega_{\mathrm{M}}/h^2 = \rho_{\mathrm{M}}/\rho_{\mathrm{crit}}$ to relate the closure density value of the matter to its actual physical density. But because $H_0^{\mathrm{FRW}} \neq H_0^{\mathrm{Obs}}$, and thus $\rho_{\mathrm{crit}}^{\mathrm{FRW}} \neq \rho_{\mathrm{crit}}^{\mathrm{Obs}}$, this ceases to be true. Specifically, $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \propto \omega_{\mathrm{M}}/(H_0^{\mathrm{FRW}})^2$ may very well be equal to unity *despite* the low value of the physical density, $\omega_{\mathrm{M}} \equiv [\rho_{\mathrm{M}}(8\pi G/3)(100 \ \mathrm{km \ s^{-1} Mpc^{-1}})^{-2}]$, when $H_0^{\mathrm{FRW}} < H_0^{\mathrm{Obs}}$ is properly taken into consideration[8]. This is important because the spatial flatness of the universe – particularly as determined using data observed from the ancient and very homogeneous CMB epoch – is dependent upon the value of the unperturbed parameter, $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, relevant to the very early universe; *not* upon the observationally-defined parameter, $\Omega_{\mathrm{M}}^{\mathrm{Obs}} \propto \omega_{\mathrm{M}}/(H_0^{\mathrm{Obs}})^2$, reflective of the more recent, post-structure-forming epoch.

To obtain an expression for the relationship between $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ and $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$ in our smoothly-inhomogeneous cosmological formalism, we begin by assuming that the universe actually is spatially flat in terms of its FRW-defined parameters, and would have appeared to be so to observers in the past, before the onset of the apparent acceleration due to clumping. Now, at such an early time, t_{E} – say, $t_{\mathrm{CMB}} \ll t_{\mathrm{E}} \ll t_0^{\mathrm{FRW}}$, for full matter-domination, but yet small inhomogeneous clumping – one had (as functions of t_{E}): $\rho_{\mathrm{M,E}} = \rho_{\mathrm{crit}}[t_{\mathrm{E}}] = 3(H^{\mathrm{FRW}}[t_{\mathrm{E}}])^2/8\pi G = (1/6\pi G)(t_{\mathrm{E}})^{-2}$. As the universe evolved to the current epoch, the volumetric dilution of matter went like $g_{rr}^{-3/2} = \{[a_{\mathrm{MD}}(t)]^{-3} \ [1 + (1/3)I(t)]^{-3/2}\}$ (cf. Equation 13), yielding a matter density today of: $\rho_{\mathrm{M,0}} = \rho_{\mathrm{M,E}} \ \{(t_{\mathrm{E}}/t_0^{\mathrm{FRW}})^2 \ [1 + (I_0/3)]^{-3/2}\} =$

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[8]Note that we assume the *physical* matter density from observations to be accurate despite causal back-reaction – i.e., $\rho_{\mathrm{M}}^{\mathrm{FRW}} \equiv \rho_{\mathrm{M}}^{\mathrm{Obs}} \equiv \rho_{\mathrm{M}}$ – since it can be measured at lower redshifts and in less cosmologically-dependent ways than Ω_{M}. Thus we treat $\omega_{\mathrm{M}}^{\mathrm{FRW}} \equiv \omega_{\mathrm{M}}^{\mathrm{Obs}} \equiv \omega_{\mathrm{M}}$ as unchanged in our formalism from the usual FLRW value, defining all of the discrepancy to be within H_0^{Obs} and $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$.

$$\{(1/6\pi G)(t_0^{\mathrm{FRW}})^{-2}\ [1+(I_0/3)]^{-3/2}\} = \{[3(H_0^{\mathrm{FRW}})^2/8\pi G]\ [1+(I_0/3)]^{-3/2}\}.$$

To turn this formula into a value for $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$, one must consider it in terms of what we *think* the critical density is today, observationally. Now clearly, $\rho_{\mathrm{crit},0}^{\mathrm{Obs}} \equiv 3(H_0^{\mathrm{Obs}})^2/8\pi G$; and simply by taking the ratio $\Omega_{\mathrm{M}}^{\mathrm{Obs}} \equiv \rho_{\mathrm{M},0}/\rho_{\mathrm{crit},0}^{\mathrm{Obs}}$, we get the following result – along with an alternative way of expressing it, including a convenient definition for $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$:

$$\Omega_{\mathrm{M}}^{\mathrm{Obs}} = (H_0^{\mathrm{FRW}}/H_0^{\mathrm{Obs}})^2\ \{[1+(I_0/3)]^{-3/2}\}\ , \tag{25a}$$

$$\Omega_{\mathrm{M}}^{\mathrm{FRW}} \equiv \Omega_{\mathrm{M}}^{\mathrm{Obs}}\ [(H_0^{\mathrm{Obs}}/H_0^{\mathrm{FRW}})^2\ \{[1+(I_0/3)]^{3/2}\}]\ \overset{!}{=} 1\ . \tag{25b}$$

Now, the proper way to view these relationships is as more of a consistency check, than as an independent prediction. If one assumes an initially-flat FRW universe, and adopts some favored inhomogeneity clumping evolution function $\Psi(t)$, then one can use it to compute I_0 (via Equation 12) and $(H_0^{\mathrm{Obs}}/H_0^{\mathrm{FRW}})$ (via Equation 23); in comparison with this, one takes the observationally measured values of the physical mass density ω_{M} and expansion rate $H_0^{\mathrm{Obs}} \equiv 100\ h^{\mathrm{Obs}}$ km s^{-1}Mpc^{-1}, and puts them together to form $\Omega_{\mathrm{M}}^{\mathrm{Obs}} = \omega_{\mathrm{M}}/(h^{\mathrm{Obs}})^2$. One then checks to make sure that Equations 25a,b are satisfied – i.e., that $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ computed numerically from the model via Equation 25a matches $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ from observations. Or equivalently (and more conveniently), as we do below, one may adopt some reliable value of $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ from observations, and put it together with I_0 and $(H_0^{\mathrm{Obs}}/H_0^{\mathrm{FRW}})$ from the model, in order to use Equation 25b to check that $\Omega_{\mathrm{M}}^{\mathrm{FRW}} = 1$. If this test is not satisfied to some acceptable level of error, then either: the early universe was not spatially flat; $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ has been poorly estimated (via measurement errors in ω_{M} and/or H_0^{Obs}); the chosen clumping model $\Psi(t)$ is not optimal; or there is a problem with the formalism itself (either fundamentally or with its simplifying approximations). The goal is therefore to find a $\Psi(t)$ that fits the supernova data well, while simultaneously achieving $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \simeq 1$ for an appropriately specified value of $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$.

One last comment about $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, is that while having it be equal to unity does represent a (primordially) 'flat' universe, this does not necessarily represent a 'critical' universe (i.e., steady expansion at a rate smoothly asymptoting to zero), since the future evolution of such a universe will strongly depend upon the detailed effects of causal backreaction. These issues will be discussed further in Section 7; though we note here that the ever-increasing strength of backreaction over time (i.e., $I(t)$ monotonically increasing towards unity for large t) would likely require the development of a fully gravitationally-nonlinear treatment of causal backreaction in order to determine the true fate of the universe.

5.3. More Key Cosmological Observables: q_0^{Obs}, w_0^{Obs}, and j_0^{Obs}

In conjunction with the fitting of cosmological evolution models to Hubble plots of the SNe data, it is also useful to extract a few standard cosmological parameters that characterize the data in a generalized way (e.g., 'accelerating' versus 'decelerating', etc.). Following Visser (2004), we give the definitions of (respectively) the Hubble, deceleration, and jerk (or jolt) functions as: $H(t) \equiv \dot{a}/a$, $q(t) \equiv -(\ddot{a}/a)H(t)^{-2}$, and $j(t) \equiv (\dddot{a}/a)H(t)^{-3}$, where $a \equiv a^{\mathrm{Obs}}(t^{\mathrm{Obs}})$ is the 'observed' volumetric scale factor (equal to $g_{rr}^{1/2}$, as could be read off from the smoothly-inhomogeneous metric, Equation 13) as a function of observable time, and overdots represent derivatives with respect to t^{Obs}.

The limiting values of these functions as $z \to 0$, $t \to t_0$ are the well-known parameters H_0^{Obs}, q_0^{Obs}, and j_0^{Obs}, which do not depend explicitly upon the entire cosmic evolutionary history, but can be described mathematically in terms of a series of Taylor expansion coefficients of the luminosity distance function, $d_{\mathrm{L}}(z^{\mathrm{Obs}})$, defined for low-$z^{\mathrm{Obs}}$. As given in Visser (2004); Riess et al. (2004):

$$d_{\mathrm{L}}(z^{\mathrm{Obs}}) = \frac{c}{H_0^{\mathrm{Obs}}}\{z^{\mathrm{Obs}}+\frac{1}{2}[1-q_0^{\mathrm{Obs}}](z^{\mathrm{Obs}})^2+\frac{1}{6}[-1+q_0^{\mathrm{Obs}}+3(q_0^{\mathrm{Obs}})^2-j_0^{\mathrm{Obs}}](z^{\mathrm{Obs}})^3+O[(z^{\mathrm{Obs}})^4]\} \,. \tag{26}$$

Extracting these cosmological parameters requires multiple derivatives of d_{L} – in our case 'differentiating' (as described above in Equations 18-19) the simulated d_{L} curve for each run.

The observed and unperturbed Hubble Constants, H_0^{Obs} and H_0^{FRW} respectively, have already been defined in terms of one another and such derivatives via Equation 23. The other cosmological parameters can be computed independently of those specific values, by taking ratios of the derivatives, as follows:

$$q_0^{\mathrm{Obs}} = 1 - \frac{d_{\mathrm{L,Pert,0}}''}{d_{\mathrm{L,Pert,0}}'} \,, \tag{27a}$$

$$w_0^{\mathrm{Obs}} \equiv \frac{2}{3}(q_0^{\mathrm{Obs}} - \frac{1}{2}) \,, \tag{27b}$$

and:

$$j_0^{\mathrm{Obs}} = -1 + q_0^{\mathrm{Obs}} + 3(q_0^{\mathrm{Obs}})^2 - \frac{d_{\mathrm{L,Pert,0}}'''}{d_{\mathrm{L,Pert,0}}'} \,. \tag{28}$$

With Equation 27b, we have also included a reference to the usual Equation of State (EoS) function, $w(z)$, which is usually interpreted (though obviously not in our formalism) as a measurement of the pressure properties of the cosmic contents, particularly that of Dark

Energy. Note, though, that this w_0^{Obs} above represents the observed EoS (or whatever effect mimics it) of the current epoch of the universe *as a whole*; it is not the same thing as a parameter characterizing the Dark Energy component *alone*, i.e., w_0^{Λ}. In particular, for a matter plus Cosmological Constant ($w_0^{\Lambda} = -1$) cosmology with a given $\Omega_{\Lambda} = (1 - \Omega_{\text{M}})$ at $z = 0$, one has $w_0^{\text{Obs}} = -\Omega_{\Lambda}$.

In terms of evaluating the third derivative of the luminosity distance function, we have deliberately chosen to characterize its behavior in terms of this jerk/jolt parameter, j_0^{Obs}, rather than using alternative formulations. This term in $d_{\text{L},0}'''$ (i.e., the $O[(z^{\text{Obs}})^3]$ term in the expansion) is the lowest-order term containing information required to characterize changes in the EoS relations of the cosmic contents over time, such as might be the result of an evolving, quintessence-like Dark Energy. Parameterizations are therefore often chosen to highlight or simplify such an analysis, by defining the Dark Energy EoS as a function of z via parameterizations such as $w^{\Lambda}(z) \equiv w_0^{\Lambda} + w^{\Lambda'}z$ (e.g., Riess et al. 2004), or $w^{\Lambda}(z) \equiv w_0^{\Lambda} + [w_{\text{a}}^{\Lambda}z/(1+z)]$ (e.g., Kowalski et al. 2008). But we do not do this here, for two important reasons.

First of all, j_0 is a purely empirical parameter, allowing our analysis to be completely agnostic with respect to the physical cause of the apparent acceleration; as our formalism does not include any form of Dark Energy, it would be less productive to use parameterizations (such as $[w_0^{\Lambda}, w_{\text{a}}^{\Lambda}]$) which are optimized to determine the EoS of a Dark Energy which is nonexistent in our models. In the language of Visser (2004), we are choosing a "retrodictive" approach, rather than a "predictive" one, by considering "cosmographic" fits without any prior assumption of Friedmann dynamics. And as has been noted in previous cosmological analyses (Cattoën & Visser 2008; Riess et al. 2007), the choice of parameterization can have a significant impact upon the best-fit results obtained, especially for data with large scatter and uncertainties.

A second useful feature of the jerk parameter, is the property that both $\Omega_{\text{M}} = 1$ SCDM *and* Cosmological Constant ΛCDM have $j(t) = j(z) = j_0 = 1$ for all time. That is, spatially flat ΛCDM models with $w^{\Lambda}(z) = -1$, containing only pressureless matter ('dust') and vacuum energy, will always have a jerk parameter of unity; a condition that will be true for any value of $\Omega_{\Lambda} = (1 - \Omega_{\text{M}})$, regardless of whether the cosmology is dust-only, dust-dominated, Λ-dominated, or Λ-only. (And presumably even a slowly-evolving Dark Energy fairly close to Λ, with $|dw(z)/dz| \ll 1$ and $w(z)$ never too far from -1, would yield $j_0 \simeq 1$.) This apparently coincidental result – it is *not* the case when significant radiation is present, for example – allows one to conduct a signature test of the entire SCDM/ΛCDM set of cosmologies, for a Dark Energy that is anything close to a Cosmological Constant. Searching for deviations from $j_0 = 1$ therefore represents an (essentially lowest-order) test of the ΛCDM

(and FLRW) paradigm itself, rather than simply narrowing down the parameters *within* that paradigm. This also represents a falsifiable test of our formalism, since our best-fit simulated cosmologies (as will be shown below) generally produce strong deviations of j_0 from unity.

5.4. Cosmic Microwave Background Observations and the CMB Acoustic Scale

Testing a theoretical model in Precision Cosmology requires the comparison of model predictions against data from several different, independent observational methods, in order to reduce the effects of large measurement uncertainties, and to disentangle parameter degeneracies. One of the most powerful probes of the universe is the Cosmic Microwave Background, so we consider here the characteristic angular scale of the CMB acoustic peaks, l_{A}, which roughly controls the positioning of the peaks in the CMB power spectrum.

Our formalism of a smoothly-inhomogeneous universe may be described as minimally-disruptive for the CMB, in that few details of the very early universe are altered from the standard FRW case: the primordial spectrum of fluctuations is not changed, and neither is the spatial flatness (on average) of the universe, the present-day physical density of matter, the ratio of baryonic matter to Dark Matter, or just about any parameter affecting the physics of the CMB epoch. The only major physical change is that made to the angular diameter distance to the last scattering surface; plus some other 'apparent' modifications due to differences between the observed (i.e., dressed) cosmological parameters, and the 'true' (i.e., bare) parameters.

The observed acoustic scale, $l_{\mathrm{A}}^{\mathrm{Obs}}$, is determined by the ratio of two values: the 'standard ruler' provided by the CMB sound horizon (r_{s}), and the angular diameter distance (d_{A}) to the last scattering surface. As discussed in Efstathiou & Bond (1999), the exact projection of a three-dimensional temperature power spectrum to a two-dimensional angular power spectrum is complicated, and depends upon the Doppler peak number m and the shape of the primordial power spectrum; but to simplify matters, we may settle here for the approximate (flat-space) relationship given in their Equation 21a: $l_m \approx m\pi d_{\mathrm{A}}/r_{\mathrm{s}}$, and thus:

$$l_{\mathrm{A}} \equiv \frac{\pi d_{\mathrm{A}}}{r_{\mathrm{s}}} \ . \tag{29}$$

Given this simplification, as well as some others (e.g., neglecting the contribution of the early ISW effect to the location of the first peak (Hu 1995), ignoring radiation in the evolution of the scale factor, etc.), our results will therefore not be directly comparable to the precise peak positions actually observed in the CMB. However, we will always be consistent in

comparisons of this l_A parameter *between* different models, including comparisons of our numerical results against the usual SCDM and ΛCDM models.

Now, for computing d_A, we note that the angular diameter distance is actually just the physical distance to that surface as would be measured at the time of the last scattering. So for flat FLRW models, the result would simply be: $d_{A,FLRW} = [a(t_{CMB})\, r_{CMB}] = [a_0\, r_{CMB}/(1+z_{CMB})]$, where r_{CMB} is the coordinate distance traveled by null rays from the CMB to us; i.e., the integral of $[c/a(t)]$ from t_{CMB} to t_0, where $a(t)$ represents whichever FLRW model one chooses.

Our inhomogeneity-perturbed model, on the other hand, requires several alterations. First, we note that the (unperturbed) scale factor, redshift and (coordinate) time of the CMB recombination will be changed. Taking z_{CMB}^{Obs} as a given measured parameter, and noting that the (pre-clumping) CMB epoch has $I(t_{CMB}) \simeq 0$, we use Equations 14-15 to get:

$$z_{CMB}^{FRW} = \frac{1 + z_{CMB}^{Obs}}{\sqrt{1 + (I_0/3)}} - 1 \; , \tag{30}$$

and hence, since $a_{MD}(t_{CMB}) \equiv a_{CMB} \equiv [a_{MD}(t_0)/(1+z_{CMB}^{FRW})]$, and $a_{MD}(t) \equiv [a_0 (t^{FRW}/t_0^{FRW})^{2/3}]$:

$$a_{CMB} = \frac{a_0}{1 + z_{CMB}^{Obs}} \sqrt{1 + (I_0/3)} \tag{31a}$$

$$t_{CMB}^{FRW} = t_0^{FRW} \left[\frac{\sqrt{1 + (I_0/3)}}{1 + z_{CMB}^{Obs}}\right]^{3/2} \; . \tag{31b}$$

Using this last result above, we can now compute the coordinate distance to the last scattering as $r^{FRW}(t_{CMB}^{FRW})$, where this value comes from an integration as defined earlier in Equation 16, given whatever clumping evolution function $\Psi(t)$ that one is doing a simulation of.

Noting finally that the physical distance to the last scattering, at t_{CMB}, is given by $[a_{CMB}\, r^{FRW}(t_{CMB}^{FRW})]$, we can now combine these previous results to get the final expression for the CMB angular diameter distance in our inhomogeneity-perturbed formalism:

$$d_{A,Pert} = a_0 \frac{\sqrt{1 + (I_0/3)}}{1 + z_{CMB}^{Obs}} r^{FRW}(t_{CMB}^{FRW}) \; , \tag{32}$$

where the arbitrary (though dimensionful) factor a_0 ultimately cancels out due to the factor of $1/a_0$ in $r(t)$, as per Equation 16c.

Now by using the purely matter-dominated (MD) expression for the evolution of the scale factor in this above calculation – e.g., using $a(t^{FRW}) \propto t^{2/3}$ to obtain Equation 31b

for $t_{\mathrm{CMB}}^{\mathrm{FRW}}$ – we have dropped the effects of radiation upon the cosmic expansion rate. We have also made this same approximation for the integration in Equation 16 for $r^{\mathrm{FRW}}(t)$, and for all other integrations and calculations in our formalism. This (greatly simplifying) approximation makes little difference for our results, since the clumping-related perturbations that we model only become important deep into the MD epoch, by which time radiation has become a fairly negligible cosmic component. The exception to this, however, is for calculations in which one goes back to very high z – such as in computing $d_{\mathrm{A,Pert}}$ for the CMB, or in quoting a total observed age of the universe. But even in those cases, the error is only around the $\sim 1\%$ level, certainly accurate enough for a proof-of-principle study in which one is comparing different paradigms against one another (as we do in this paper), as opposed to a high-precision analysis attempting to extract best-fit cosmological parameters from the data.

Next, to calculate r_{s} for spatially flat FLRW models, Efstathiou & Bond (1999) give the expression:

$$r_{\mathrm{s,FLRW}} = \frac{c}{\sqrt{3}} \, \frac{1}{H_0\sqrt{\Omega_{\mathrm{M}}}} \, \frac{1}{1+z_{\mathrm{CMB}}} \int_0^{a_{\mathrm{CMB}}} \frac{d(a/a_0)}{\{[(a/a_0) + (a_{\mathrm{eq}}/a_0)] \, (1+R)\}^{1/2}} \, , \qquad (33)$$

where a_{eq} represents the scale factor at equality between matter and radiation (the latter including nearly massless neutrinos), and $R \equiv (3\rho_{\mathrm{b}}/4\rho_\gamma) \propto (a/a_0)$ is the ratio determining the sound speed of the photon-baryon fluid via $c_{\mathrm{s}} = [(c/\sqrt{3}) \, (1+R)^{-1/2}]$ (Hu & Sugiyama 1995). Using numerical estimates for the present-day densities and properties of radiation and neutrinos, they then give the values: $(a_{\mathrm{eq,FLRW}}/a_0) = (24185 \, \omega_{\mathrm{M}})^{-1}$ (for three light neutrino flavors), and $R_{\mathrm{FLRW}} = [30496 \, \omega_{\mathrm{b}} \, (a/a_0)]$; and it becomes a simple matter to perform the integration and calculate $r_{\mathrm{s,FLRW}}$.

For our model, once again, several modifications must be made. Note first that we have already made two alterations in Equation 33 from the precise formula given in Efstathiou & Bond (1999), which will be needed for clarity in our following calculations. First, we do not implicitly normalize all FLRW scale factors (e.g., a_{eq}, a_{CMB}) to $a_0 = 1$, but rather normalize them explicitly (when necessary) as a/a_0, etc.; this is necessary because $a(t) \neq \sqrt{g_{rr}(t)}$ in our formalism when $I(t) \neq 0$, so that setting $a_0 = 1$ no longer properly normalizes $g_{rr}(t_0)$ to unity. And second, while their r_{s} is typically called "the sound horizon at decoupling", what it actually is, is the sound horizon at decoupling measured *today*. To convert it to the sound horizon size *then* (for proper comparison to d_{A} above), we have had to multiply it by $(a_{\mathrm{CMB,FLRW}}/a_{0,\mathrm{FLRW}}) = (1 + z_{\mathrm{CMB,FLRW}})^{-1}$.

In order to generalize these results for our formalism, all of these ratios of scale factors have to be modified to account for inhomogeneity-induced perturbations. Once again we

have $a_{\mathrm{CMB}} = [a_0 \sqrt{1 + (I_0/3)}/(1 + z_{\mathrm{CMB}}^{\mathrm{Obs}})]$ as per Equation 31a, and we can similarly write:

$$a_{\mathrm{eq}} = \frac{a_0}{1 + z_{\mathrm{eq}}^{\mathrm{Obs}}} \sqrt{1 + (I_0/3)} , \qquad (34)$$

where $z_{\mathrm{eq}}^{\mathrm{Obs}}$ is the usual redshift of equality computed from the observed densities according to the FLRW formalism.

Relatedly, the baryon-to-photon ratio as a function of scale factor will need to be adjusted. Noting that R will evolve here in precisely the same way as it does in the FLRW formalism when measured as a function of evolving z^{Obs} (rather than a or t), we write:

$$R_{\mathrm{Pert}}(t^{\mathrm{FRW}}) \equiv R_0 \, \frac{a}{a_0} \, \frac{\sqrt{1 + [I(t^{\mathrm{FRW}})/3]}}{\sqrt{1 + (I_0/3)}} , \qquad (35)$$

to represent the proper volumetric dilution, where the present-day baryon-to-photon ratio (assumed fixed by observations) factors in here as $R_0 \equiv (3\rho_{\mathrm{b},0}/4\rho_{\gamma,0})$. We must include the factor $\sqrt{1 + [I(t^{\mathrm{FRW}})/3]}$ in the formal definition above to get the correct value of R_{Pert} at t_0^{FRW}, though it reduces to unity in these CMB-related calculations because $I(t \leq t_{\mathrm{init}}) \equiv 0$ in our models (justifiable since $I(t_{\mathrm{CMB}}) \simeq I(t_{\mathrm{eq}}) \simeq 0$ in the real universe).

With these relations and definitions, we can integrate from scratch to find the sound horizon using $c_{\mathrm{s,Pert}} = [(c/\sqrt{3}) \, (1 + R_{\mathrm{Pert}})^{-1/2}]$, along with the normal evolution of a radiation/matter early (FRW) universe. Keeping careful track of all perturbation factors, we get the result:

$$r_{\mathrm{s,Pert}} = \frac{c}{\sqrt{3}} \, \frac{1}{H_0^{\mathrm{FRW}}} \, \frac{\sqrt{1 + (I_0/3)}}{1 + z_{\mathrm{CMB}}^{\mathrm{Obs}}} \quad \times$$

$$\int_0^{a_{\mathrm{CMB}}} \{ (\frac{a}{a_0} + \frac{\sqrt{1 + (I_0/3)}}{1 + z_{\mathrm{eq}}^{\mathrm{Obs}}}) \, [1 + (\frac{a}{a_0})(\frac{R_0}{\sqrt{1 + (I_0/3)}})] \}^{-1/2} \, d(\frac{a}{a_0}) . \qquad (36)$$

Note that expressing the prefactor of the integral in terms of H_0^{FRW}, as opposed to H_0^{Obs}, automatically eliminates the $\Omega_{\mathrm{M}}^{-1/2}$ factor (i.e., $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$ is implicitly set to unity, as required for our spatially flat dust-only model).

Inserting the results of Equation 36 (which can be integrated numerically) for $r_{\mathrm{s,Pert}}$ and Equation 32 for $d_{\mathrm{A,Pert}}$ into Equation 29, we finally obtain an expression for $l_{\mathrm{A}}^{\mathrm{Obs}} = (\pi \, d_{\mathrm{A,Pert}}/r_{\mathrm{s,Pert}})$ to be calculated for each of our simulation run outputs. To get numerical results, however, one must assign values to the parameters $z_{\mathrm{eq}}^{\mathrm{Obs}}$, $z_{\mathrm{CMB}}^{\mathrm{Obs}}$, and $R_0 = [(3/4) \, (\omega_{\mathrm{b},0}/\omega_{\gamma,0})] = [(3/4) \, (\omega_{\mathrm{b},0}/\omega_{\mathrm{M},0}) \, (1 + z_{\mathrm{eq}}^{\mathrm{Obs}})]$. (Note that the direct dependence upon $H_0^{\mathrm{FRW}} \propto 1/t_0^{\mathrm{FRW}}$ cancels out of the ratio $(d_{\mathrm{A,Pert}}/r_{\mathrm{s}})$; cf. Equations 16c, 17c, 32, 36.)

Using the best-fit (WMAP-only data) cosmological parameters given in Hinshaw et al. (2009), we have $z_{\mathrm{eq}}^{\mathrm{Obs}} = 3176$, $z_{\mathrm{CMB}}^{\mathrm{Obs}} = 1090.51$, and $(\omega_{\mathrm{b},0}/\omega_{\mathrm{M},0}) = [\omega_{\mathrm{b},0}/(\omega_{\mathrm{CDM},0} + \omega_{\mathrm{b},0})] =$

(0.02273/0.13263). We take these as actual measurements of physical observables, which are not altered by our smoothly-perturbed universe model; rather, it is the theoretical ("FRW") model parameters which must be modified, in order to match these observed values.

To compare the l_A^{Obs} values from our models to those of unperturbed FLRW cosmologies, we use a flat ΛCDM cosmology that is best-fit to real SNe data. Further discussion of this best-fit optimization step is given below, in Section 5.5.

In order to construct a FRW SCDM model to compare these CMB (and other) calculations to, we increase Ω_M from ~ 0.27 to 1.0, we multiply $\omega_{b,0}$, $\omega_{M,0}$, $(1 + z_{\text{eq}}^{\text{Obs}})$ (and hence R_0) by $(1.0/0.27)$, and we leave $(1 + z_{\text{CMB}}^{\text{Obs}})$ unchanged.

With all of these parameters and formulae, we can now calculate the CMB acoustic scale l_A^{Obs} (and all of the other cosmological parameters specified previously) for each of our numerically-simulated cosmologies, and compare them to the (homogeneous) ΛCDM and SCDM cases. These results will be collected and discussed soon below, in Section 6.1.

One last remark about the calculations in this subsection, though, is that one must not overestimate their precision in estimating CMB observables. There is a great distance in time and space between our era and the decoupling epoch, and there will be many effects in a universe described by an inhomogeneity-perturbed formalism like ours (e.g., possible modifications to lensing, to the ISW effect, etc.), that are not included in these above calculations. Such complex effects are useful, since they should ultimately help in distinguishing our formalism from FLRW/Dark Energy models; but for now, it must be noted that the preceding formulae for observed cosmological parameters – and especially these calculations of the CMB acoustic scale – are being derived from a highly simplified and averaged model of the universe.

5.5. Computing χ^2 Values and Fit Probabilities for our Inhomogeneity-Perturbed Hubble Diagrams

To quantitatively assess how well our simulated cosmological models (plotted earlier in Figures 3-5) manage to reproduce the apparent acceleration, we must analyze how well they fit a sample of reliable supernova data. The publicly available "SCP Union" compilation (Kowalski et al. 2008) of 307 SNe Ia (after selection cuts) will serve as the fiducial set of supernovae for all of the analyses in this paper, except where stated otherwise.

The output of each of our numerically-simulated models can be converted into a distance modulus function via $\mu_{\text{Pert}} \equiv (m - M)_{\text{Pert}} = \{5 \, \text{Log}_{10}[d_{\text{L,Pert}}(z^{\text{Obs}})] + 25\}$, with $d_{\text{L,Pert}}(z^{\text{Obs}})$

computed according to Equation 17. It can then be turned into a residual distance modulus function, $\Delta\mu_{\mathrm{Pert}} \equiv \Delta(m - M)_{\mathrm{Pert}}$, by subtracting off a coasting universe model (as per Equation 21) for analysis and for plotting in residual Hubble diagrams.

Now since this formula for $\Delta\mu_{\mathrm{Pert}}$ is auto-normalized to give a zero y-intercept as $z \to 0$, there is no need to specify a particular Hubble Constant in the theoretical model. Real data, on the other hand, must be normalized by assigning some value of H_0 to the coasting universe model that gets subtracted from the data in order to compute the set of $\Delta\mu_{\mathrm{SN}} \equiv \Delta(m - M)_{\mathrm{SN}}$ values. (Generally speaking, there will always be an adjustable factor H_0^{Obs} needed in order to create a comparison between a model, which does not have any specific Hubble Constant intrinsic to it, versus the observed data, which does.) Riess et al. (2004) state that the chosen value of H_0 (equivalent to the absolute distance scale) is arbitrary, since it only shifts the plot of $\Delta(m - M)$ up or down by a constant amount, and that their analysis only depends upon *differences* in magnitude. In practice, however, choosing a poor value of H_0 not only makes χ^2 worse for every model, but there is also no guarantee that the *relative* goodness-of-fit between different models will stay the same when this constant offset is changed, particularly in the case of data with large uncertainties and scatter. Furthermore, each different theoretical model requires a somewhat different vertical offset (i.e., its own individualized H_0^{Obs} value) in order for that specific model, and the dynamical cosmological evolution that it represents, to fit a particular SN data set as well as it possibly can. In our analysis, therefore, we optimize H_0^{Obs} separately for each simulation run (done simply by adding, and optimizing, a constant offset value to $\Delta\mu_{\mathrm{SN}}$, for its comparison to that particular $\Delta\mu_{\mathrm{Pert}}$ curve). Such optimization of the observed Hubble Constant is naturally subject to external constraints due to other astronomical observations which limit its acceptable range of values; but this is in fact advantageous, since the best-fit value of H_0^{Obs} that is output for each specific simulation run gives us one more independent check upon whether or not that model with those input parameters is an acceptable approximation of reality, above and beyond its ability to properly fit the SNe data.

One other challenge in comparing our numerical models to the SNe data is that our simulated results for d_{L} are only lists of points for discrete values of z^{Obs}, not continuous functions. To evaluate differences between μ_{Pert} and μ_{SN} for any given z_{SN}, some form of interpolation is necessary to make μ_{Pert} continuous. Since our simulations use enough points to sample z very finely – with, for example, inter-pixel gaps of $\Delta z \lesssim 10^{-3} - 10^{-2}$ for $z^{\mathrm{Obs}} \lesssim 1 - 3$, and $\Delta z \lesssim$ few $\times 10^{-4}$ for $z^{\mathrm{Obs}} \lesssim 0.1$ – any decent interpolation scheme should give good results. In all of our runs, we compare three different functions for interpolating between data points, using polynomials of order 1, 2, and 3 in z^{Obs}; and we find that these three interpolation schemes always produce χ^2 values that are identical to one another for at least 5 or 6 significant figures.

Given such a continuous interpolation function $\mu_{\mathrm{Pert}}(z^{\mathrm{Obs}})$ for a particular simulation run, we can compute the χ^2 value for that theoretical model fit as follows:

$$\chi^2_{\mathrm{Fit}} = \sum_i \frac{[\mu_{\mathrm{SN},i} - \mu_{\mathrm{Pert}}(z^{\mathrm{Obs}}_{\mathrm{SN},i})]^2}{\sigma^2_{\mathrm{SN},i}} \; , \tag{37}$$

where $\mu_{\mathrm{SN},i}$ and $\sigma_{\mathrm{SN},i}$ for each SN are as given in Kowalski et al. (2008) and associated SCP data files. (Note that we do not separately fold in additional SNe dispersions due to peculiar velocities, lensing, or other intrinsic or systematic effects, as they do in order to be conservative in estimating ranges of cosmological parameter values; such additions here would simply make it harder to distinguish between the quality of different theoretical fits, without providing any new useful information.)

Once this χ^2_{Fit} value (optimized with respect to H_0^{Obs}) has been calculated for a given inhomogeneity-perturbed model, one may compute the likelihood of this μ_{Pert} curve by integrating the cumulative distribution function for the χ^2 distribution with N_{DoF} degrees of freedom (i.e., $\chi^2_{N_{\mathrm{DoF}}}[X]$), as follows:

$$P_{\mathrm{Fit}} \equiv 1 - P_{N_{\mathrm{DoF}}}\{0 \leq X \leq \chi^2_{\mathrm{Fit}}\} = 1 - \int_0^{\chi^2_{\mathrm{Fit}}} \chi^2_{N_{\mathrm{DoF}}}[X] \; dX \; . \tag{38}$$

This P_{Fit} represents the goodness-of-fit probability that the given theoretical curve, if it actually is a correct description of the universe, would give a value of χ^2 as high (or higher) than the χ^2_{Fit} that was found. (Although we will see that the P_{Fit} values calculated for the models considered here – including best-fit ΛCDM – are relatively small ($\sim 0.3 - 0.4$), they are not small enough to be a serious concern, since adding in the necessary systematic uncertainties not already folded into these $\sigma_{\mathrm{SN},i}$ values would make all of the fit probabilities larger.)

The relevant number of degrees of freedom here is given by $N_{\mathrm{DoF}} = (N_{\mathrm{SN}} - N_{\mathrm{Fit}})$, where $N_{\mathrm{SN}} = 307$ for the SCP Union data set, and N_{Fit} is the number of model fitting parameters, once a particular type of theoretical model has been chosen. Now first of all, H_0^{Obs} being individually optimized for all theoretical and simulated Hubble curves gives us one fitting parameter for every model. Additionally: flat ΛCDM has $N_{\mathrm{Fit}} = 2$, since only the value of Ω_Λ (along with H_0^{Obs}) is optimizable; and flat SCDM, with no remaining adjustable parameters, has $N_{\mathrm{Fit}} = 1$. Alternatively, our Ψ_{Lin}, Ψ_{Sqr}, and Ψ_{MD} models each have $N_{\mathrm{Fit}} = 3$, since both Ψ_0 and z_{init} can be varied for fitting the data. Thus the number of degrees of freedom for evaluating the likelihoods of our inhomogeneity-perturbed models is $N_{\mathrm{DoF}} = 304$; whereas the "Concordance" ΛCDM fit ($\Omega_\Lambda \simeq 0.73 = 1 - \Omega_{\mathrm{M}}$) has $N_{\mathrm{DoF}} = 305$, and flat SCDM has $N_{\mathrm{DoF}} = 306$.

One thing that we do *not* do in this analysis, however, is to compute the "reduced χ^2", $\chi^2_{\mathrm{Fit,DoF}} \equiv (\chi^2_{\mathrm{Fit}}/N_{\mathrm{DoF}})$. While it is quite common (e.g., Riess et al. (2004), and many other sources) to use $\chi^2_{\mathrm{Fit,DoF}} \sim 1$ as a proxy for indicating a good model fit to the data, it is a poor statistical practice for informally estimating likelihoods. The use of $\chi^2_{\mathrm{Fit,DoF}}$ implicitly assumes the approximation, $P_1\{0 \leq X \leq \chi^2_{\mathrm{Fit,DoF}}\} \simeq P_{N_{\mathrm{DoF}}}\{0 \leq X \leq \chi^2_{\mathrm{Fit}}\}$. But $\chi^2_1[\chi^2_{\mathrm{Fit}}/N_{\mathrm{DoF}}]$ is a very poor representation of $\chi^2_{N_{\mathrm{DoF}}}[\chi^2_{\mathrm{Fit}}]$ for large N_{DoF}, because the former distribution is much broader and more gradual: it has a longer right tail, thus leading to a much larger chance of a Type II error (incorrect acceptance of a false hypothesis, e.g., Ross (1987)); and the probability distribution increases more slowly as one goes towards lower χ^2 values, thus also leading to a larger chance of a Type I error (incorrect rejection of a true hypothesis). In particular, in the case of the fitting results to be given below for our best simulation runs and for the ΛCDM model, in which the results span $\chi^2_{\mathrm{Fit,DoF}} \sim 1.02 - 1.05$ for $N_{\mathrm{DoF}} \sim 304 - 305$, the use of the "reduced" $\chi^2_{\mathrm{Fit,DoF}}$ would cause us to unknowingly mis-estimate model-fitting probabilities of $P_{\mathrm{Fit}} \sim 26\% - 38\%$ as probabilities of $P_{\mathrm{Fit}} \sim 30\% - 31\%$ – a significant loss of *comparative* information about how well the different models fit the data. In this paper, therefore, we will stick to the more accurate statistical practice of simply quoting χ^2_{Fit}, N_{DoF}, and the resulting P_{Fit} for each theoretical or simulated model.

6. OBSERVATIONAL TESTS OF THE FORMALISM

In order to firmly establish our smoothly-inhomogeneous backreaction formalism as an acceptable paradigm for understanding the cosmic evolution, we must ultimately accomplish three goals: (1) Explaining the 'already-known' – i.e., reproducing the most important observational results that have formerly been interpreted as signs of Dark Energy; (2) Providing 'falsifiability' for our formalism, by establishing *new* predictions that can clearly distinguish our model from the conventional ΛCDM Concordance paradigm; and lastly: (3) Convincingly argue plausibility or 'naturalness', in the sense of showing that our model does not suffer from similar fine-tuning problems as the Dark Energy approach. For this third point, in particular, while the linking of the onset of the apparent acceleration with the beginning of widespread structure formation clearly removes the Coincidence Problem of Cosmological Constant Dark Energy (recall Section 1), it still remains to be shown *why* an alternative, non-Λ explanation of the cosmic acceleration, such as ours – even if true – should happen 'fortuitously' to look so much like the action of a trivially simple (if aesthetically and coincidentally unpleasant) Cosmological Constant.

The first two of these three tasks will be discussed below, in Subsections 6.1 and 6.2, respectively. The third task – addressing the basic necessity, itself, of employing an alternative

cosmology in the face of current cosmological data sets that appear to be fairly consistent with the ΛCDM Concordance Model – will be reserved for a future treatment, both because of space limitations here, and due to the fact that this is a generic question for all alternative cosmologies, not a unique issue for the causal backreaction formalism introduced in this paper. We will therefore present our detailed discussion on that topic elsewhere; but in any case, it is likely that all of these above issues cannot be resolved to a satisfactory degree without much more comprehensive and precise cosmological data, along with continued input from the broader expertise of the entire cosmological community.

6.1. Supernova Fits and Cosmological Parameters from our Numerical Simulations: Evaluating the New Concordance

Using the fit probabilities, P_{Fit}, obtainable from each of the simulation runs presented earlier in Section 4, in conjunction with the cosmological parameters calculated from each run according to the formulae in Section 5, we now have a quantitative context for judging our causal backreaction paradigm and the clumping evolution functions $\Psi(t)$ that have been modeled for this paper. Given those results, we have specified an informally-chosen set of 'best' runs for further detailed discussion here. Of the sixty cosmological models plotted previously in Figures 3-5, twelve of them (six Ψ_{Lin} runs and six Ψ_{MD} runs) appear to this author as being 'very good' at replicating the apparently accelerating behavior of the universe, while also having fairly good cosmological parameters.

Now, given that these so-called "best runs" are chosen simply from a discrete set of 60 runs performed overall, it would seem likely that a truly optimized search over the $(\Psi_0, z_{\text{init}})$ parameter space might find model runs that do an even better job at fitting the SNe data. Such an effort is not really called for here, though, given the theoretical uncertainties of our formalism – particularly the neglect of both gravitational and recursive nonlinearities, as well as the ad-hoc nature of our chosen $\Psi(t)$ clumping functions, themselves – but such an optimization can indeed be done, producing many fits with values of χ^2_{Fit} that are just as low (and often slightly lower) than that of the best-fit flat ΛCDM model. This is most naturally accomplished by choosing parameters to weaken the Ψ_{MD} runs, since strengthening the 'accelerative' effects of the Ψ_{Lin} or Ψ_{Sqr} runs requires one to implausibly resort to $z_{\text{init}} \gg 25$ or $\Psi_0 > 1$. Exploring the range of models with Ψ_{MD} clumping evolution, there turns out to be an extensive 'trench' in parameter space extending at least from $(\Psi_0, z_{\text{init}}) = (0.89, 5)$ to $(0.765, 14)$ (and likely beyond) for which $\chi^2_{\text{Fit}} \lesssim 311.7$ always remains true, allowing one to fit the SNe data with these models slightly better than one can with ΛCDM over a wide range of model input parameters, thus simultaneously providing good fits and opening up an

extended range of output cosmological parameters to more flexibly match the model results to other external observational constraints.

For illustration purposes, we choose one of these input-parameter-optimized runs as an additional example model for comparing to the SNe data and observed cosmological parameters. All of the Ψ_{MD} runs in this phase-space trench are virtually degenerate in their χ^2_{Fit} values (there is an extremely weak gradient in P_{Fit} among them), so as a somewhat loosely defined "semi-optimized" run we select the $(\Psi_0, z_{\mathrm{init}}) = (0.768, 14)$ case, which has the lowest χ^2_{Fit} of any Ψ_{MD} run that we found among those tested (in a non-exhaustive search through the parameter space), while also having sufficiently different parameters from our discrete sample of 20 Ψ_{MD} runs for it to be of further cosmological interest. We thus add this case to our twelve other "best runs" discussed above, giving us a set of thirteen highlighted runs, in total, for use in the more in-depth discussion of the results of our simulations which follows now.

Residual Hubble diagrams of these thirteen chosen runs are shown in Figure 6, plotted along with the (now SNe-best-fit) SCDM and Concordance ΛCDM models. Shown against these curves are the SCP Union SNe data, with the 307 SNe averaged here into 40 bins, equally spaced in $\mathrm{Log}[1 + z^{\mathrm{Obs}}]$; binning is necessary for clear visualization of this data set, given not only the large number of SNe data points, but also the very high scatter for those data, and the large error bars for each individual supernova magnitude.

It is obvious by inspection that these thirteen runs represent cosmological models that produce good Hubble curves, being visually almost indistinguishable from one another (and from Concordance ΛCDM) in the SNe-data-rich region of $z^{\mathrm{Obs}} \sim 0.1 - 1$. This is especially evident when plotted on this y-axis scale; a scale that clearly shows the large separation between all of these models from SCDM, as well as the large scatter and error bars of the SNe data themselves – *even after* binning and averaging – when contrasted with the tight overlap between all of these models and the Concordance ΛCDM cosmology itself.

For a more quantitative analysis, the comprehensive output data for these thirteen runs, along with corresponding output parameters from the best-fit ΛCDM and SCDM runs, are given in Table 1.

First, considering the goodness-of-fit of the models to the SNe data, we see that the results for this discrete selection of inhomogeneity-perturbed models are nearly as good as that for ΛCDM in terms of χ^2_{Fit}; and they are comparable in terms of P_{Fit}. Our thirteen 'best' models range from $\chi^2_{\mathrm{Fit,Pert}} \sim 311.7 - 319.0$, with the "semi-optimized" run (and others near it in the 'preferred Ψ_{MD} parameter space trench' discussed above) having $\chi^2_{\mathrm{Fit,Pert}} < \chi^2_{\mathrm{Fit,\Lambda CDM}} = 311.9$; and the P_{Fit} values for most of these thirteen models are almost as good

as the Concordance ΛCDM result here, with ten of them having $P_{\mathrm{Fit,Pert}} > 0.3$, and five of those going as high as $P_{\mathrm{Fit,Pert}} \sim 0.36 - 0.37$ (as compared to $P_{\mathrm{Fit,\Lambda CDM}} = 0.38$ for the fully-optimized case of $\Omega_\Lambda = 0.713$, $H_0 = 69.96$ km s^{-1}Mpc^{-1}). In short, it seems fair to conclude that the smoothly-inhomogeneous formalism presented in this paper – even at this 'toy model' stage, given all of the simplifications discussed above – is already able to successfully reproduce the apparent acceleration of the universe essentially as well as the more standard ΛCDM paradigm can do.

Next, to check on the general validity of these models all the way to $t = t_0$, we must consider the diagnostic parameter $I_0 \equiv I(t_0)$, which varies from $I_0 \sim 0.52 - 0.72$ for these thirteen best runs, and stays bounded at $I_0 \leq 0.66$ for eleven of them. As discussed in Section 3.2, where $I(t)$ was formally constructed, this function (representing the causally-integrated influence of inhomogeneities) is only valid in the linear context of Newtonian gravitational terms, for which the metric perturbations from different masses can be simply summed together. To the extent that $I(t)$ approaches unity, the linearized-gravity approach of our formalism breaks down, and full general relativity becomes necessary for describing the combined effect of all perturbations, in total.

Now, seeing I_0 take on values roughly midway between 0 and 1 for these runs is in fact a good sign for trusting the general predictions of these models. If I_0 (and thus $d[I(t)]/dt$ as well) were too small, then the total perturbative effect of these inhomogeneities on the average cosmic gravitational potential would be too weak to cause much of an observable effect at all (recall Equations 4a,b and the related discussion) – certainly not a reaction strong enough to explain the apparent acceleration. For example, the very weak FRW-perturbing behaviors of the Ψ_{Sqr} models (cf. Figures 4), which have $I_0 \lesssim 0.2$, is symptomatic of their limited accelerative effects (i.e., $w_0^{\mathrm{Obs}} > -0.5$). On the other hand, too large a value – such as $I_0 \simeq 0.93$, from the strongest of all of our models (the Ψ_{MD} case with $(\Psi_0, z_{\mathrm{init}}) = (1.0, 25)$) – is so large that the detailed quantitative results of such a model cannot really be trusted.

The fact that the perturbative effects from our thirteen best-fitting model cosmologies do not completely overshoot the linearity approximation, and thus remain fairly trustworthy in their output results, is not completely a matter of luck: one key feature of the cosmic acceleration (and the source of the Coincidence Problem) is that it seems to have just recently begun, being due (in the backreaction paradigm) to the same structure formation that recently (in cosmological terms) has created us, as well. But this 'luck' will not hold out in the long term, since $I(t)$ should continue to grow in the future for many billions of years, until $I(t \gg t_0)$ eventually becomes so large that the evolution of the real universe itself (and not just our phenomenological representation of it) should cease to be describable by any simply-perturbed FRW/FLRW model. The dynamical behavior of such a universe

would likely become quite extreme and difficult to predict, and we will speculate upon the possible cosmic futures in Section 7.

Moving on to the observable cosmological parameters for each run, we first note the changes to z^{Obs}, so that it is now no longer precisely equal to z^{FRW}. Table 1 gives results corresponding to $z^{\text{FRW}} = 1$ (i.e., $[a_0/a(t)] = 2$) as a sample epoch, demonstrating that while the differences are not huge (e.g., $z^{\text{Obs}} \sim 11 - 14\%$ larger than z^{FRW} for these thirteen runs), they are also non-negligible, implying that the backreaction effects of inhomogeneities have a real ability to alter the relationships between observed cosmological time, volume, and redshift, which must be taken into account for any precision understanding of the evolution of astrophysical objects.

The effects upon the Hubble Constant, on the other hand, are much larger in magnitude, as we have both expected and required, in order to create an alternative concordance. First, considering the individually-optimized values of the observed Hubble Constant for each run, we see that our thirteen best runs (along with the flat ΛCDM model) have a range of $H_0^{\text{Obs}} \sim 68.8 - 71.8$, which compares well with the result of $H_0 = 70.1 \pm 1.3$ km s^{-1}Mpc^{-1} from combined (WMAP+BAO+SN) data (Hinshaw et al. 2009). This is an encouraging cross-check of these runs, in contrast to the utter failure of $\Omega_{\text{M}} = 1$ SCDM (with optimized $H_0^{\text{Obs}} = 61.35$) to properly fit either the SNe data or the Hubble Constant; similar to the tendency of our model simulation runs with poor SNe P_{Fit} values (not presented here) to *also* have observationally discrepant H_0^{Obs} values.

Next, considering the unperturbed/"FRW" Hubble Constant values for these thirteen best runs, we get results of $H_0^{\text{FRW}} \sim 34.2 - 42.8$, with six of the very best values lying within $H_0^{\text{FRW}} \sim 36.1 - 41.6$. These results are entirely in accord with our discussion from Section 5.1, in which we noted how the findings of other authors have indicated that a variety of other cosmological measurements are seemingly concordant with $\Omega_{\text{M}} = 1$ models that have a low Hubble Constant, only to be stymied by direct observations apparently indicating $H_0 \sim 70$. But here we see that the ability of our inhomogeneity-perturbed formalism to achieve a value of H_0^{Obs} in the ~ 70's, despite having a 'true' Hubble Constant – i.e., the one that actually mattered during the pre-structure-formation cosmic epoch – of H_0^{FRW} in the ~ 30's-40's, completely removes this contradiction. Furthermore, it also helps in the establishment of an alternative, matter-only concordance in several other ways.

One important, related aspect of this low value of H_0^{FRW} is that we have now *solved the Age Problem* in consequence, as described above in Section 5.1, without needing a period of recent 'real' acceleration that would nominally be provided by Dark Energy. Our results for these thirteen runs are $t_0^{\text{Obs}} \simeq 13.2 - 14.5$ GYr (and $t_0^{\text{Obs}} \simeq 13.4 - 14.2$ GYr for the nine best fits), which compares very favorably to the SN-best-fit flat ΛCDM value of 13.64 GYr

(noting again that the time estimates here are not exact, since none of these models include radiation; though they are all mutually consistent for $\Omega_R \equiv 0$). Now, a cosmic age difference of a $\sim$billion years either way is certainly not negligible, and careful observations of objects like globular clusters, etc., could be expected to someday strongly discriminate between ΛCDM and our formalism (as well to as help optimize parameters *within* our formalism, for different clumping evolution models). But given the typical state of cosmic age lower-limits, taken from the measured ages of astrophysical objects – for example, like that from a review on Cosmic Age in Spergel et al. (2003), where the strongest lower limit that they quoted was 12.7 ± 0.7 GYr, from observations of cooling White Dwarfs – it seems reasonable to conclude that the values given here for t_0^{Obs} from our formalism, even at this 'toy model' stage, are good enough to have effectively solved the Age Problem possessed by SCDM; especially considering that $t_{0,\text{SCDM}}$, in contrast, is as low as $\sim$10.6 GYr, as seen in Table 1.

Given H_0^{FRW} (along with H_0^{Obs} and I_0) for each run, we can also now address the question of the spatial flatness of the universe, by computing Ω_M^{FRW} via Equation 25b. This first requires us to choose some value of Ω_M^{Obs} from cosmological observations. Now, this value need not be equal to $(1 - 0.713) = 0.287$, which one might infer from the SN-best-fit flat ΛCDM model above, since it would be better to use a value that comes from a more comprehensive combination of different data sets; and a value of $\Omega_M^{\text{Obs}} \simeq 0.287$ would in fact be rather high, given most recent estimates of Ω_M^{Obs}.

The actual amount of matter as a fraction of the apparent (i.e., observational) closure density is still fairly difficult to pin down with great precision. Spergel et al. (2007) demonstrated in their Tables 5 and 6 that which best-fit value of Ω_M^{Obs} they obtained depended upon which external data set (if any) that they chose to combine the (3rd-Year) WMAP data set with, with variations of $\Omega_M \sim 0.226 - 0.299$ possible. Dunkley et al. (2009) noted the tension in the preferred (high versus low) values of the matter density observed by SDSS ($\Omega_M = 0.265 \pm 0.3$), and 2dFGRS ($\Omega_M = 0.236 \pm 0.02$). And while they described how the uncertainty in the matter density had dropped with each new analysis of the growing WMAP data set, it is also true that there has been some oscillation in those estimated values, with the 1st-Year WMAP analysis giving $\Omega_M = 0.27 \pm 0.04$ (Bennett et al. 2003), the 3-Year WMAP mean dropping to $\Omega_M = 0.241 \pm 0.034$, and the 5-Year WMAP mean rising back up (despite a max likelihood of only 0.249) to $\Omega_M = 0.258 \pm 0.03$ (Dunkley et al. 2009, Table 2). Furthermore, that 5-Year WMAP-only result actually increases to $\Omega_M = 1 - 0.721 = 0.279$ (Hinshaw et al. 2009, Table 6) when complementary data sets are included (WMAP+BAO+SN). Similarly, Larson et al. (2011) give $(\Omega_c + \Omega_b) \simeq 0.274$ for their (updated) 7-Year WMAP values.

For the calculations of Ω_M^{FRW} in this paper, we choose to adopt what seems to be a generally reasonable value for our search for a new concordance: $\Omega_M^{\text{Obs}} \equiv 0.27$. What must

be realized in these calculations of the 'true' (pre-perturbation) spatial curvature $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, however, is that although we have adopted one specific value of $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$ for quoting values of $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, we can only trust our estimates of spatial flatness (i.e., $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \to 1$) as being accurate to within the observational uncertainties in $\Omega_{\mathrm{M}}^{\mathrm{Obs}}$; for example, to within a (very approximate) range of, say, $\sim (1/0.27) \cdot [\pm 0.1] \simeq \pm [0.3 - 0.4]$, or so.

Looking at the $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$ results for our thirteen best runs in Table 1, we can clearly say that to within these uncertainties, our smoothly-inhomogeneous cosmological formalism has indeed achieved spatial flatness; and that this has been done without requiring the incorporation of any "missing mass" or "missing energy" (beyond the usual Cold/Hot Dark Matter), such as Dark Energy, to bring Ω_{Tot} up to unity.

Specifically, we see that these Ψ_{Lin} runs yield $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \sim 0.90 - 1.04$, and that the Ψ_{MD} runs yield $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \sim 0.99 - 1.64$ (narrowing down further to $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \sim 0.99 - 1.41$, if we drop the worst of these seven Ψ_{MD} cases). Thus the Ψ_{Lin} runs are especially good at achieving flatness. And the Ψ_{MD} runs, even with their stronger effects, are not too far off either: if one (reasonably, as we have seen) chooses a lower value of the observed matter density – say, $\Omega_{\mathrm{M}}^{\mathrm{Obs}} = 0.24$, instead of 0.27 – then this drops the range down to $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \sim 0.88 - 1.25$ for the six best Ψ_{MD} runs, making them even more consistent with $\Omega_{\mathrm{M}}^{\mathrm{FRW}} \equiv \Omega_{\mathrm{Tot}} = 1$. Even given the observational uncertainties, as well as the many theoretical simplifications of our formalism, it is a notable step towards the achievement of an alternative concordance that such a trial-and-error set of sixty simulated cosmological models (with astrophysically-motivated parameters) has been found to include about a dozen runs that succeed in bringing $\Omega_{\mathrm{Tot}}^{\mathrm{FRW}}$ from ~ 0.3 up to a reasonably close range around 1, using matter alone.

Next, we consider the quantitative amount of apparent acceleration – i.e., the degree to which $q_0^{\mathrm{Obs}} < 0$, $w_0^{\mathrm{Obs}} < (-1/3)$ – that our models have produced. Table 1 shows that each of these thirteen best runs have produced a *strong* amount of 'acceleration', with $w_0^{\mathrm{Obs}} \sim (-0.71) - (-0.82)$ [i.e., $q_0^{\mathrm{Obs}} \sim (-0.56) - (-0.73)$] for the Ψ_{Lin} runs; and $w_0^{\mathrm{Obs}} \sim (-0.75) - (-1.0)$ [i.e., $q_0^{\mathrm{Obs}} \sim (-0.62) - (-1.0)$] for the Ψ_{MD} runs.

Now, a flat ΛCDM universe with a given value of (Cosmological Constant) Ω_{Λ} will have $w_0^{\mathrm{Obs}} = -\Omega_{\Lambda}$. Taking the values of $\Omega_{\Lambda} = 0.742 \pm 0.03$ (WMAP-only) and $\Omega_{\Lambda} = 0.721 \pm 0.015$ (WMAP+BAO+SN) from Hinshaw et al. (2009) as a guide, it seems like we would look for our models to reproduce values in the range of, say, $w_0^{\mathrm{Obs}} \sim (-0.70) - (-0.77)$, in order to look like Concordance ΛCDM. As we see, four of these Ψ_{Lin} runs, and one of these Ψ_{MD} runs, do in fact fall within this range, with the others producing somewhat stronger 'acceleration' effects.

While this is already a fairly good success rate for our models in reproducing the appar-

ent acceleration, it is important to note that the precise value of w_0^{Obs} which is 'observed' is actually just the result of some particular best-fit procedure, given some assumed cosmological model; and output parameters from best-fits depend non-trivially upon the fitting function used. For example, Kowalski et al. (2008) quote a mean value (from SN+BAO+CMB data) of $w_0^\Lambda = -0.969$ for a constant-w^Λ analysis, which in conjunction with their $\Omega_\Lambda = 0.713$, turns into $w_0^{\mathrm{Obs}} = (0.713) \cdot (-0.969) = -0.691$. Alternatively, in their varying Dark Energy EoS analysis using $w^\Lambda(z) \equiv w_0^\Lambda + [w_{\mathrm{a}}^\Lambda z/(1+z)]$, they get (Rubin, D. 2008, private communication) a mean value of $w_0^\Lambda = -1.13$ (with $w_{\mathrm{a}}^\Lambda = 0.73$), which in conjunction with a re-fit value of $\Omega_\Lambda = 0.718$, yields $w_0^{\mathrm{Obs}} = -0.811$; this is clearly a substantial change from $w_0^{\mathrm{Obs}} = -0.691$, just given a change in the fitting assumptions. The conclusion here is that it is not necessary to *precisely* match the amount of 'acceleration' (i.e., w_0^{Obs}, $w^{\mathrm{Obs}}(z)$) found with the best-fit Dark Energy models, in order to say that one has reproduced the apparent cosmic acceleration. Rather, all that one has to do is to produce an apparent acceleration that is very roughly in the correct range of the old Concordance w_0^{Obs}, while simultaneously producing a fit to the SNe data that is essentially as good or better than ΛCDM. As seen above, we have achieved this by obtaining $\sim 5-10$ runs here with fits of a quality comparable to that of ΛCDM, with nearly all of them having an apparent acceleration within the fairly good range of $w_0^{\mathrm{Obs}} \sim (-0.7) - (-0.9)$.

One further way of characterizing the cosmic acceleration, in particular its 'sudden' onset, is to describe the universe as having recently experienced a strong, positive "jerk" (Riess et al. 2004). Given our definition of the jerk/jolt parameter in Equation 28, we find our six Ψ_{Lin} runs in Table 1 to have $j_0^{\mathrm{Obs}} \sim 2.5 - 3.5$, while the seven Ψ_{MD} runs have $j_0^{\mathrm{Obs}} \sim 2.6 - 5.5$. Recalling from Section 5.3 that spatially flat cases of both SCDM and Cosmological Constant ΛCDM models (for any value of $\Omega_\Lambda = 1 - \Omega_{\mathrm{M}}$) *always* have $j_0^{\mathrm{Obs}} = 1$, it thus appears that these universally high values of j_0^{Obs} – universal, that is, for those of our models also capable of fitting the SNe data and w_0^{Obs} properly – is the result representing *the most discriminating test* that we have found so far which could potentially distinguish our formalism from traditional Concordance Models with a Cosmological Constant (or anything close to it) that are capable of generating the apparent acceleration. Given that the quantity j_0^{Obs} – or its corresponding alternative in 'dynamic' parameterizations, w_{a}^Λ – comes from the third-derivative term in the expansion for d_{L} in z^{Obs} (cf. Equation 26), its value is not yet observationally well constrained. In that sense, we will regard the output values of j_0^{Obs} from our runs as a *prediction* of our formalism, rather than as an attempt to match known results; a matter which we will explore further in Section 6.2.2 below, where we consider (in brief) the observational situation for j_0^{Obs} as it stands.

Unfortunately, though measurements of $j_0^{\mathrm{Obs}} > 1$ would conclusively rule out ΛCDM, we cannot conclude with certainty, based only upon these above results, that high jerk

parameters are a robust feature of our formalism. Too many theoretical uncertainties exist within our models to make this an 'iron-clad' prediction of our calculations just yet. A number of different effects should all act to moderate the strength of the apparent acceleration as $z \to 0$, $t \to t_0$ – thus weakening the late-time "cosmic jerk" – both within our model calculations and in the real universe.

First, there is the issue of "recursive nonlinearities" discussed earlier in Section 3.2: the fact the pre-existing inhomogeneities serve to slow down the causal updating which brings in later information about new inhomogeneities, thus softening the apparent acceleration effect by some undetermined degree. As calculating these effects properly would take an algorithmically nonlinear simulation program, we cannot quantitatively estimate them here; all that we can say is that they would provide some significant amount of damping of the apparent acceleration effects due to causal backreaction, especially at later times.

Second, even if we could safely rely upon this and other simplifications in our formalism, and even if the functional forms of the Ψ_{Lin} and/or Ψ_{MD} clumping evolution models (cf. Equations 22a,b) generally do make sense for the long-term growth of structure in the universe, there is still no guarantee that those cluster evolution functions would remain meaningful all the way to $t \to t_0$. As structure forms, the large-scale collapse of material and feedback from star formation act to shock heat cosmic baryons to millions of degrees, particularly at late times ($z^{\mathrm{Obs}} \lesssim 3$), thus inhibiting clumping[9] by keeping and/or sending a significant portion of the baryons into the superheated IGM (e.g., Cen & Ostriker 2006). Now, our models can account for *some* of this effect, simply by using a lower value of Ψ_0; but if this shock-heating is strong enough to really soften the evolution of growth of cosmic structure (thus significantly altering the functional form of $\Psi(t)$, itself), then this would be a moderating effect that is not taken account of by any of the (highly simplified) clumping evolution functions that we use in this paper.

Any of these effects which weaken the apparent acceleration as $z \to 0$ are generally bad for our Ψ_{Lin} models, since this could make at least some of them too weak to produce a new cosmic concordance; alternatively, they are generally good for our Ψ_{MD} models, by reducing their computed values of $(H_0^{\mathrm{Obs}}/H_0^{\mathrm{FRW}})$, $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, and w_0^{Obs}. In any case, however, such effects

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[9]On the other hand, given that simulations (Ho et al. 2006) show that galaxy cluster ellipticities are still decreasing as $z \to 0$ due to continued relaxation, this means that at least some virialization does continue all the way to the current time, even for structures which largely are 'already clumped' – thus implying a continuing "backreaction" of the kind which we phenomenologically model here. But whether or not this continuing level of virialization would be strong enough to substantially counter the slowing of such backreaction at late times due to shock heating, it would in any case represent yet more complexity in determining the most physically realistic functional form to use for $\Psi(t)$.

would likely lead to a reduction in our calculated output values of j_0^{Obs}. The magnitude of such a change is not certain, since the primary effect of such alterations might simply be to change *which* choices of model input parameters (i.e., which $(\Psi_0, z_{\text{init}})$ and/or which type of $\Psi(t)$ function) manages to fit the cosmological data best, *without* changing the general values of j_0^{Obs} (or of other output parameters) that tend to emerge from those models that do achieve good SNe fits. But all we can say for certain here, examining Table 1 once more, is that for the seven runs 'most acceptable' on every front – i.e., those runs with $\chi^2_{\text{Fit}} < 315$, $t_0^{\text{Obs}} \sim 13.4 - 14.0$ GYr, $\Omega_{\text{M}}^{\text{FRW}} \sim 0.97 - 1.3$, and $w_0^{\text{Obs}} \sim (-0.75) - (-0.87)$ – that we have the range of $j_0^{\text{Obs}} \sim 2.6 - 3.8$ for the jerk parameter. It therefore seems reasonable, at this stage, to propose that such a range of j_0^{Obs} may be a generic prediction from our formalism for models which succeed in achieving a good alternative concordance – a conclusion subject, of course, to further theoretical development of this paradigm.

Lastly in our evaluation of these thirteen best model runs, we consider the acoustic scale of the CMB peaks, $l_{\text{A}}^{\text{Obs}}$, as derived above in Section 5.4. As noted there, this parameter is the one incorporating the most far-ranging assumptions and simplifications, and the one most prone to error due to the great look-back time and distance to the last scattering surface. The theoretical uncertainties in our results for $l_{\text{A}}^{\text{Obs}}$ are therefore expected to be large, though we cannot precisely quantify them here.

Table 1 shows that these six best Ψ_{Lin} runs have $l_{\text{A}}^{\text{Obs}} \sim 284.2 - 291.3$, while the six best Ψ_{MD} runs (dropping the worst of these Ψ_{MD} SNe fits) have $l_{\text{A}}^{\text{Obs}} \sim 270.5 - 288.7$. Narrowing it down even further, the six runs most acceptable on every front (i.e., the seven 'most acceptable' runs referred to previously, now minus the "Semi-Optimized" Ψ_{MD} run) have $l_{\text{A}}^{\text{Obs}} \sim 277.5 - 288.7$. Those values succeed very well in bracketing the SN-best-fit ΛCDM value (for no radiation) of $l_{\text{A},\Lambda\text{CDM}}^{\text{Obs}} = 285.4$, while simultaneously producing good values for all of the other cosmological parameters computed in this paper, as well.

Such results, while serving as a fairly good match of our models to the CMB acoustic scale, may not quite match the precision of the actual $\pm \sim 0.8 - 0.9$ measurement uncertainties quoted in observational CMB parameter estimation results for $l_{\text{A}}^{\text{Obs}}$ (e.g., Hinshaw et al. 2009). Nevertheless, it is significant to note how much better these estimates are than any fit achievable with a low-Ω_{M} *open* CDM model without Dark Energy. For example, using the rough rule-of-thumb (Turner 1999) of $l_1^{\text{Obs}} \simeq 200 \, \Omega_{\text{Tot}}^{-1/2}$ for the first CMB peak, shifting from a flat SCDM universe ($\Omega_{\text{M}} = 1$) to an oCDM universe ($\Omega_{\text{M}} \sim 0.3$, $\Omega_\Lambda \equiv 0$) increases l_1^{Obs} by $\sim 200[(1/\sqrt{0.3}) - 1] \simeq 165 \equiv \Delta l_{\text{A}}^{\text{Obs}}$. Keeping $\Omega_{\text{M}}^{\text{Obs}} \sim 0.3$ without some mechanism to bring about spatial flatness – such as Dark Energy, or our smoothly-inhomogeneous perturbation formalism – therefore leads to errors in $l_{\text{A}}^{\text{Obs}}$ of *over a hundred;* whereas insisting on flat SCDM mostly saves $l_{\text{A}}^{\text{Obs}}$ (see Table 1), but at the cost of ruining the fit to the SNe data

($P_{\mathrm{Fit}} \simeq 3.4 \times 10^{-22}$). Our formalism, on the other hand, preserves the good SNe fit (and good values for all of the other cosmological parameters calculated here), while producing deviations from the Concordance ΛCDM model of only $\Delta l_{\mathrm{A}}^{\mathrm{Obs}} \lesssim 15$ for twelve of these thirteen best runs (and to within $\Delta l_{\mathrm{A}}^{\mathrm{Obs}} \sim 0.2 - 7.9$ for the six very best runs), all while maintaining $\Omega_{\Lambda} \equiv 0$.

To sum up the results of this subsection: from the basic set of sixty simulation runs done with our inhomogeneity-perturbed formalism using astrophysically-motivated input parameters, we have obtained twelve runs that produce quantitatively good fits to residual Hubble diagrams of the SCP Union supernovae, many of them very close in χ^2_{Fit} to the best-fit done with flat ΛCDM. In addition to these, we include a thirteenth run, taken as one example of a long trench in $(\Psi_0, z_{\mathrm{init}})$-space containing many Ψ_{MD} runs which produce nearly equal values of χ^2_{Fit}, all lower than that achievable with any flat ΛCDM model. These thirteen-plus causal backreaction models have therefore successfully reproduced the signs of apparent cosmic acceleration that are seen in Type Ia SNe data sets. Furthermore, as it is not surprising that good fits to the SNe data also imply good values for other parameters, we see that six of these runs – specifically, Ψ_{Lin} with $(\Psi_0, z_{\mathrm{init}}) = (1.0, 25)$, $(1.0, 15)$, and $(0.96, 25)$, and Ψ_{MD} with $(\Psi_0, z_{\mathrm{init}}) = (0.78, 10)$, $(0.85, 5)$, and $(0.92, 5)^{10}$ – not only provide excellent SNe fits, but also reproduce several other cosmological parameters to within acceptable 'Concordance-level' bounds, including t_0^{Obs}, $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$, w_0^{Obs}, and $l_{\mathrm{A}}^{\mathrm{Obs}}$. And beyond just achieving an agreement for already-known observable parameters with these models, we also find them to yield $j_0^{\mathrm{Obs}} \sim 2.6 - 3.8$, indicating that observations of a jerk parameter value significantly greater than unity would as of now appear (pending further theoretical refinements of these calculations) to serve as a potentially powerful way of distinguishing our formalism from Cosmological Constant ΛCDM, as we will examine further next.

We note once again that all of this is achieved in our models *without* the incorporation of any Dark-Energy-type (i.e., negative pressure) species, but is instead done simply with a spatially flat, matter-dominated universe that is perturbed by causally-propagating information about self-stabilizing inhomogeneities.

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[10]Recalling once more that larger z_{init} and mid-range Ψ_0 are the most 'astrophysically sensible' values for these parameters, generally speaking.

6.2.   Distinguishing the Smoothly-Inhomogeneous Formalism from Concordance ΛCDM

A candidate paradigm for an alternative Cosmic Concordance, such as our formalism, must simultaneously satisfy a variety of fairly independent observational constraints, including the parameters considered above plus several other types of observations.

Assuming that such a goal can be fully achieved, it becomes rather difficult then to find distinctions which would be capable of demonstrating a statistically strong preference for the alternative concordance over one achievable with Dark Energy (DE); especially so in the case of data with high scatter and uncertainties, and a paradigm as malleable as DE with an optimizable equation of state. Nevertheless, we will discuss a few potential methods here to distinguish our formalism from Dark Energy in general, where possible; and more feasibly, from Cosmological Constant DE in particular, which despite being aesthetically questionable and more constrained (and thus easier to falsify) than evolving DE, still appears consistent with a wide range of observations.

6.2.1.   Direct and Indirect Effects of Inhomogeneities: Observational Anisotropies and Other Cosmological Signatures

One obvious departure of our formalism from any non-clustering version of DE is the importance that our model places upon the combined influence of many localized inhomogeneities. Though our calculations are done using a "smoothly" inhomogeneous ansatz in which spatial variations are averaged away, in the real universe these effects will not perfectly average to smoothness, and thus it becomes more interesting in our paradigm to look for evidence of unexpectedly large anisotropies on a variety of scales.

Though there seems to be no substantial rejection yet of the large-scale homogeneity assumed by the Cosmological Principle and typical FLRW cosmologies, some intriguing observational results have turned up using various methodologies. A preliminary study by this author (Bochner 2007b) of anisotropies in the Riess gold04 SNe compilation (Riess et al. 2004) showed some marginal, positive signs of the existence of real anisotropies; and studies using SNe, galaxy clusters, etc., to map anisotropies (or find enhanced variances) in the Hubble Flow by other researchers – e.g., Kolatt & Lahav (2001); McClure & Dyer (2007); Schwarz & Weinhorst (2007); Jain et al. (2007); Blomqvist et al. (2008); Seikel & Schwarz (2009); Colin et al. (2010) – more or less show similar results, finding signs of anisotropy of varying statistical significance.

Alternatively, considering inhomogeneities seen via the CMB, there have been stud-

ies concerning the possibility of significant anisotropies (Hansen et al. 2004; Bernui et al. 2007), non-Gaussianities (McEwen et al. 2005; Tojeiro et al. 2006), and other unexpected features (e.g., Larson & Wandelt 2004) present in the WMAP data (Hinshaw et al. 2007; Dunkley et al. 2009), such as the low CMB quadrupole (e.g., Jarosik et al. 2010), the CMB Axis of Evil (Land & Magueijo 2005), and the WMAP Cold Spot (e.g., Cruz et al. 2007). Also, some CMB glitches (e.g., Hunt & Sarkar 2007), though wiped out by binning (Hinshaw et al. 2007), may perhaps be signs of real inhomogeneity effects that should not be averaged away. (Though one must not ignore counterarguments (e.g., Bennett et al. 2010) that anomalies such as these mentioned here might in large part be due to *a posteriori* selection effects and technological limitations in the observations.)

Still more directly, one may investigate major cosmological structures themselves in detail, such as the Shapley Concentration and/or Great Attractor regions (e.g., Bardelli et al. 1994) and the Sloan Great Wall (Gott et al. 2005), and use them as part of the effort to map the detailed inhomogeneity and velocity flow structure of the universe on cosmologically large scales (e.g., Lucey et al. 2005; Bolejko & Hellaby 2008; Colin et al. 2010). Such studies are particularly interesting, given the evidence of a possible "Dark Flow" (Kashlinsky et al. 2008, 2010) which may be due to a tilt across our entire current observational horizon (if not due to some causal-inhomogeneity-driven mechanism), as well as other evidence of bulk flows out to $100h^{-1}$Mpc or more (Feldman et al. 2010).

Searches like these for cosmic inhomogeneities and/or anisotropies are already fully underway through the work of many researchers, as a way of determining the fine details of the cosmic evolution. Our work here merely serves to add to the impetus behind such investigations, given our proposal that inhomogeneities may very well determine the *average* observed cosmological parameters themselves, not just the *deviations* from the averaged, so-called best-fits.

Even without direct evidence of inhomogeneities, though, the method of apparent cosmic acceleration that we propose here would undoubtedly impose changes upon several other cosmological measurements, changes which we have not explicitly estimated in this paper. Particularly interesting are observations which are not explained by (or which directly conflict with) the ΛCDM paradigm. For example, the fact that causal backreaction does not occur in completely smoothly-distributed fashion, but is instead concentrated mostly near virializing masses, may lead to interesting clustering-induced feedback behaviors affecting the formation of stars, galaxies and galaxy clusters. Conceivably, this may have some useful application to issues such as galaxy downsizing (Cowie et al. 1996), the cuspy CDM halo problem, and the possible dearth of dwarf satellite galaxies (Primack 2003); but such connections are speculative, and require a detailed quantitative analysis to see if causal backreaction truly

succeeds (or significantly helps) in explaining these issues.

Other examples of interesting processes or observables which might be affected by causal backreaction include the Late ISW effect (e.g., Ho et al. 2008); the shape parameter, $\Gamma \equiv \Omega_M h$ (Turner 1999); observations of peculiar velocities constraining $\sigma_8 \Omega_M^{0.6}$ (Lahav & Liddle 2008); large-scale velocity flows constraining $\beta \equiv \Omega_M^{0.6}/b$, with b as the linear bias factor for galaxies versus the overall matter distribution (Fukugita & Hogan 2000; Fukugita 2001); observations of weak lensing due to large-scale structure, constraining $\sim \sigma_8 \Omega_M^{0.53-0.64}$, depending upon the angular scales used (Fu et al. 2008); Baryon Acoustic Oscillations, which constrain the combination $A \equiv \{\sqrt{\Omega_M} \, [H_0/H(z_*)]^{1/3}[r_*/z_*]^{2/3}\}$, with r_* being the dimensionless comoving distance to sampled data having typical redshift z_* (Marassi et al. 2010); indications of acceleration from the Alcock-Paczyński test (Alcock & Paczyński 1979; Marinoni & Buzzi 2010); and observations of the growth and evolution of large galaxy clusters, analyzed in conjunction with several other types of measurements (e.g., Mantz et al. 2008; Vikhlinin et al. 2009).

A serious study of how all of these (and other) observations would be modified by our apparent acceleration mechanism goes far beyond the scope of this paper, and would likely be premature with these calculations being essentially in the toy model stage. But any eventual finding that our formalism does as well (or better) than simple Dark Energy models at achieving a concordance, based upon a variety of such measurements, would be convincing evidence in favor of our paradigm over the more physically exotic FLRW approach.

6.2.2. Testing Our Formalism with Estimates of the Jerk Parameter, j_0^{Obs}

Given the importance of the jerk parameter in potentially falsifying ΛCDM (via $j_0 \neq 1$) and supporting our formalism (if j_0 significantly exceeds unity), it is useful to provide an estimate of the actual value of j_0 here. Accurately measuring j_0 is quite difficult, however, even with the best of today's Precision Cosmology data sets, as one is required to go to third order in the expansion of the luminosity distance (cf. Equation 26) for cosmographic studies; or equivalently, one must go to fits with four terms, $(H_0, \Omega_\Lambda, w_0^\Lambda, w_a^\Lambda)$, in dynamical studies with an evolving Dark Energy EoS, in order to calculate j_0 as a function of $(\Omega_\Lambda, w_0^\Lambda, w_a^\Lambda)$. The difficulty of producing stable, precise estimates of j_0 and higher-order cosmological parameters is evident in Xu & Wang (2010), in which they use their own cosmographic methods to obtain values like $j_0 = -4.996^{+7.0293}_{-7.331}$ and $j_0 = 15.665^{+59.715}_{-33.812}$, for two cases using different combinations of external data sets in conjunction with the recent SCP Union2 SNe compilation (Amanullah et al. 2010).

Several previous estimates of j_0 by this author and by other researchers (details not given here), obtained from different SNe data sets and produced with a variety of methods, show a general tendency for $j_0 > 1$ (ranging broadly from $j_0 \sim 0.9 - 5.5$ in most cases); and indeed most data sets and analysis methods give values for j_0 on the high side – though there are important exceptions, such as the Constitution SNe compilation (Hicken et al. 2009a), which prefers the rather low value of $j_0 \sim 0.5$. This latter result reflects the significant statistical differences between the first SCP Union SNe compilation (Kowalski et al. 2008) and the Constitution set; for the former, most of the allowed parameter space (see their Figure 16) prefers the Phantom Energy regime ($w_0^\Lambda < -1$), whereas for the latter, Figure 4 of Hicken et al. (2009b) shows the Constitution set to be more centered within the Quintessence region ($w_0^\Lambda > -1$) of DE parameter space. (Conditions of $w_0^\Lambda < -1$ tend to prefer $j_0 > 1$, barring a reversal due to large values of w_a^Λ of the wrong sign; and vice-versa for $w_0^\Lambda > -1$ favoring $j_0 < 1$.)

The Constitution papers themselves (Hicken et al. 2009a,b) do not an include an analysis of DE with a time-varying EoS; and here we will focus upon SNe from the original SCP Union compilation (our benchmark data set for cosmological model fits), along with the more recent SCP Union2 compilation (which supersedes the original Union and Constitution data sets), using a couple of straightforward methods for calculating j_0.

As our theoretical motivation is to estimate the purely observable parameter j_0^{Obs} devoid of any assumptions about a DE equation of state, the most direct, cosmographic approach is to do polynomial best-fits to the SNe luminosity distance values. Using the Taylor series expansion for $d_{\text{L}}(z^{\text{Obs}})$ (cf. Equation 26), it is straightforward to invert the coefficients from such a polynomial best-fit in order to obtain values for H_0^{Obs}, q_0^{Obs} (or equivalently w_0^{Obs}, cf. Equation 27b) and j_0^{Obs} from the first three terms in the expansion.

The main difficulty, as explored in a key paper (Cattoën & Visser 2008) examining earlier SNe data sets, is that such estimates of cosmological parameters (even for the second-order term q_0^{Obs}) are "distressingly" unstable, depending very sensitively upon which redshift-distance relation one selects (the luminosity distance, d_{L}, is not a unique choice), which redshift variable one uses, and how many terms one employs in the best-fit polynomial. This parameter instability (even when the polynomial expansions used all provide quantitatively very good fits) renders standard uncertainty estimates useless, as we will have to dismiss cases with 'unrealistic' parameters out of hand, based upon subjective prior judgments of likely ranges for w_0^{Obs} and j_0^{Obs}.

First, considering the most standard case – polynomial fits to $d_{\text{L}}(z)$ – it is obvious from Equation 26 that we must include at least three terms (i.e., up to $O[(z)^3]$) in order to obtain a value for j_0. But that does not mean that we must use *only* three terms; in fact, we could

include any number of terms, $N \geq 3$, in our expansion, and use only the first three of them to obtain a value of j_0 calculated independently of those higher-order terms (thus disregarding their weak information regarding higher-order cosmological observables). The advantage of using $N > 3$ expansion terms is that this gives the best-fit polynomial additional terms for modeling the high-z behavior of the data, without forcing that constraining responsibility onto the third term from which we must calculate j_0. As put by D. Rubin (2008, private communication), the $N = 3$ case "truncates the series expansion of a(t) after j_0, so even a LCDM universe will not give a fit of $j_0 = 1$"; a problem alleviated via $N > 3$. The downside of using too many terms, however, is that they become far too weakly constrained by the data, making the effects of statistical uncertainties worse (Cattoën & Visser 2008), so that even empirically good fits will yield more and more unrealistic cosmological parameters for larger and larger N. In practice, we perform polynomial fits using $N = (3, 4, 5, 6)$ terms, with the $N = (3, 4)$ cases usually producing the best parameter values (and often the only sensible ones).

Applying these polynomial fits to $d_L(z)$ for the original Union SNe compilation, we obtain $(w_0^{Obs}, j_0^{Obs}) = (-0.746, 1.32)$ for the $N = 3$ case, and $(w_0^{Obs}, j_0^{Obs}) = (-0.822, 2.49)$ for the $N = 4$ case. Things appear to start going bad for the $N = (5, 6)$ cases (with $(w_0^{Obs}, j_0^{Obs}) = (-0.625, -1.76)$ and $(-0.918, 7.19)$, respectively); but the $N = (3, 4)$ cases make a range of $j_0^{Obs} \sim 1.3 - 2.5$ (or more broadly, $\sim 1 - 3$) for the best-fit values seem quite reasonable. This is in decent accord with the varying-w analysis in (Kowalski et al. 2008), where the allowed parameter space depicted in their Figure 16 shows the ΛCDM point (i.e., $(w_0, w_a^\Lambda) = (-1, 0)$, equivalent to $j_0 = 1$) to lie just around the inside edge of the 1σ statistical ellipse.

Applying these same fits to $d_L(z)$ for the Union2 SNe compilation, one gets a startling result: the $N = 3$ case produces $(w_0^{Obs}, j_0^{Obs}) = (-0.729, 0.991)$, and the $N = 4$ case yields $(w_0^{Obs}, j_0^{Obs}) = (-0.733, 1.050)$. (With the $N = (5, 6)$ cases again being less physically reasonable, yielding $(w_0^{Obs}, j_0^{Obs}) = (-0.697, 0.187)$ and $(-0.419, -8.619)$, respectively.) Such estimates of $j_0^{Obs} = 0.991$ and 1.050 would appear to be amazingly good for Cosmological Constant DE, seemingly virtually to prove ΛCDM. But in fact, while these results are most definitely *consistent* with ΛCDM, they are misleadingly precise – there is no way that the Union2 SNe data alone could reliably yield estimates of j_0^{Obs} to within 5% or less of the ΛCDM value, even if that model is true. Such values are impossibly accurate, requiring us to dig a little deeper in order to get some sense of the real parameter estimate uncertainties.

Following Cattoën & Visser (2008), we note that $d_L(z)$ is not really a good function to use for a Taylor series expansion. First of all, the series will not even converge for $z \gtrsim 1$ (a relevant redshift range for at least ~ 20 of the Union2 SNe), rendering the high-z

behaviors of the best-fit polynomials meaningless. Rather, they recommend the use of the "y-redshift" variable, $y \equiv z/(1 + z) = [1 - (a/a_0)]$, which is bounded above by unity for past epochs – i.e., $y \in [0, 1)$ for $z \in [0, \infty)$. Second, in order to remove the dominating influence of the "nuisance" parameter, H_0^{Obs}, which does not encode any of the dynamical cosmological information, they recommend instead fitting to a function such as $\ln[d_{\mathrm{L}}(y)/y]$. Series expansions of this type of function have a first term (the only one containing the Hubble Constant) that is decoupled from the higher terms and contains no powers of y, and can thus be subtracted off as a constant offset. Lastly, they note that there is nothing necessarily unique or special about the luminosity distance function, d_{L}; and that several other cosmological distance functions – e.g., the "photon count distance" d_{P}, the "angular diameter distance" d_{A}, etc. – can be obtained from it simply by dividing by different powers of $(1 + z) = 1/(1 - y)$. This is troublesome because fitting data with large scatter and uncertainties (such as the SNe data) to different distance functions will produce extremely different values of (q_0, j_0) – especially for fits to polynomials with a small number of terms, N – and because (as they claim), "There is no good physics reason for preferring any one of these distance variables over the others."

For our analysis here, we will consider fits of the Union2 SNe to $\ln[d_{\mathrm{P}}(y)/y]$, where $d_{\mathrm{P}} \equiv d_{\mathrm{L}}/(1+z) = d_{\mathrm{L}}(1-y)$. We use d_{P} for three reasons. First, we argue that $d_{\mathrm{P}} = (a_0 \, r_{\mathrm{SN}})$ actually happens to be the most physically appropriate distance function, being simply equal to the present physical distance from the SN to us; it is the only distance function without an (artificial) factor of $(1 + z)$ in its definition, and thus it will be finite for any observed physical object (even back to the Big Bang, assuming finite cosmological horizons), and goes to zero as $z \to 0$ simply as the real distance to such an object would become zero. Second, as d_{P} is in the middle of the five physically-motivated distance functions considered by Cattoën & Visser (2008), it yields the median values of their parameter estimates, and quoting the median estimates that one may find would seem to be a conservative strategy. Third, as a practical matter, we find that none of our polynomial fits to $\ln[d_{\mathrm{L}}(y)/y]$ yield reasonable values for j_0^{Obs} (not to mention w_0^{Obs}); whereas our $N = 3$ fit (though only that one) to $\ln[d_{\mathrm{P}}(y)/y]$ is more well-behaved.

The result of this $N = 3$, $\ln[d_{\mathrm{P}}(y)/y]$ fit to the Union2 data set is $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.750, 1.338)$. This value of j_0^{Obs} is also consistent with ΛCDM, while being less unbelievably accurate, and gives us some (very qualitative) sense of the uncertainties involved. Simultaneously, it remains slightly high of ΛCDM, as our formalism predicts; and if not nearly so high as those values of j_0^{Obs} from our simulated cosmologies given above in Table 1, the incorporation of recursive nonlinearities into our models will probably bring their calculated values of j_0^{Obs} somewhat lower, as discussed previously. Nevertheless, Cosmological Constant ΛCDM remains entirely consistent with these results.

One other issue for these cosmological parameter fits relates to the "outlier rejection" process performed on these SNe data compilations, done as a final supernova cut, after all other cuts based upon the internal quality of the data points themselves have been completed. This outlier rejection cut is the removal of SNe data points based upon their poor ($\geq 3\sigma$) fits to flat ΛCDM reference cosmologies that are individually optimized and applied to each of the component data sets that go into the Union and Union2 compilations. Extensive arguments are given in Kowalski et al. (2008) and Amanullah et al. (2010) justifying this outlier rejection process as an important technique for robust statistical analysis; and yet, since the reference cosmologies used for the cuts are invariably the same type of flat ΛCDM models that are at the heart of the Cosmic Concordance Model, there is the unavoidable possibility that such outlier rejection cuts may remove some legitimate evidence against ΛCDM, perhaps thereby introducing some pro-Concordance bias into the results. It is useful here to provide a simple demonstration of the extent to which cosmological parameter estimates can be dependent upon the decisions made regarding such data cuts.

This 3σ outlier rejection cut removed 8 SNe from the SCP Union compilation, reducing the number of SNe to 307 (i.e., culling $\sim 2.5\%$ of the data); and removed 12 SNe from the Union2 compilation, reducing it to 557 SNe (i.e., culling $\sim 2\%$ of the data). (Furthermore, Amanullah et al. (2010) applied a 5σ 'outlier cut' earlier on in their analysis, which had already removed $\sim 6\%$ of their data; but it is not clear if this is the same type of cut as the cosmology-dependent outlier rejection done at the end.) Here we specifically consider the Union2 compilation with the 12 'outlier' SNe added back in (i.e., 569 SNe in total). Above, without these outliers, our $N = 3$ fit to $d_{\mathrm{L}}(z)$ yielded $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.729, 0.991)$; adding the outliers back in[11], this changes to $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.736, 1.151)$. Similarly, the $N = 4$ fit to $d_{\mathrm{L}}(z)$ changes from $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.733, 1.050)$ to $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.826, 2.633)$. (Also, the $N = 5$ fit to $d_{\mathrm{L}}(z)$ now gives sensible results, $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.775, 1.351)$.) Meanwhile, the $N = 3$ fit to $\ln[d_{\mathrm{P}}(y)/y]$ changes from $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.750, 1.338)$ to $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}}) = (-0.810, 2.422)$. Adding the 12 Union2 outliers back in thus moves the best-fit j_0^{Obs} results much farther away from the ΛCDM value of $j_0 = 1$ than before (though under the circumstances, not representing a statistically convincing rejection). What this may ultimately prove is not immediately clear, since it is readily apparent from inspection of the wild scatter of the outlier SNe points that the quality of those measurements is

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[11]The twelve outlier points, like the rest of the Union2 SNe data, are publicly available at http://supernova.lbl.gov/Union; but the outlier points are only available in a table including all SNe with lower precision values and uncertainties, and with a different H_0 normalization. Going to lower precision for the standard Union2 compilation of 557 SNe only changes the cosmological parameter estimates from polynomial fitting by $\sim 0.3 - 5.5\%$, so for consistency, we do our entire Union2+Outliers analysis (569 SNe) using the lower-precision data table.

quite mixed, and several of them undoubtedly really are bad data points; but it does show how sensitive these cosmological parameter estimates can be to decisions made about data handling that are nearly invisible in the final quoted results.

Now, these j_0^{Obs} estimates from polynomial fits to the SNe data alone, while interesting, undoubtedly have uncertainties large enough to seriously limit their usefulness. In order to get more precise estimates of j_0^{Obs} – and to be able to calculate some believable number for the statistical uncertainty on that value – we are compelled to combine the SNe data with complementary cosmological data sets (e.g., SN+CMB+BAO). It is not trivial to do this in a completely cosmographic manner, and so we therefore make the concession of considering j_0 in light of the evolving-EoS Dark Energy analyses done in the Union and Union2 papers.

For these analyses, the SCP collaboration adopts the form:

$$w^{\Lambda}(z) \equiv w_0^{\Lambda} + w_{\mathrm{a}}^{\Lambda} \frac{z}{1+z} = w_0^{\Lambda} + w_{\mathrm{a}}^{\Lambda} y \ , \tag{39}$$

widely referred to as the Chevallier-Polarski-Linder (CPL) parameterization (Chevallier & Polarski 2001; Linder 2003) for the Dark Energy EoS.

Using the cosmic expansionary history that would result from such a Dark Energy (Linder 2003) to relate these CPL parameters to the cosmographic parameters $(w_0^{\mathrm{Obs}}, j_0^{\mathrm{Obs}})$ obtained from the series expansion of $d_{\mathrm{L}}(z)$ (cf. Equation 26), a straightforward calculation yields:

$$w_0^{\mathrm{Obs}} = w_0^{\Lambda} \ \Omega_{\Lambda} \ , \tag{40}$$

which makes obvious sense; and:

$$j_0^{\mathrm{Obs}} = \{1 + [\frac{9}{2}\Omega_{\Lambda}w_0^{\Lambda}(1 + w_0^{\Lambda})] + [\frac{3}{2}\Omega_{\Lambda}w_{\mathrm{a}}^{\Lambda}]\} \ , \tag{41}$$

which gives $j_0^{\mathrm{Obs}} = 1$ for the Cosmological Constant case of $(w_0^{\Lambda} = -1, w_{\mathrm{a}}^{\Lambda} = 0)$, for *any* value of Ω_{Λ}, as required. Furthermore, note that Equation 41 is an *exact* expression for j_0^{Obs} in terms of $(\Omega_{\Lambda}, w_0^{\Lambda}, w_{\mathrm{a}}^{\Lambda})$, given how they are all defined, even though it has been isolated by doing a comparison of various terms between two different series expansions.

For use in applying this to the SCP Union compilation study in Kowalski et al. (2008), D. Rubin states (2008, private communication) that their best-fit parameter values for the CPL Dark Energy EoS, in a combined analysis of Union SNe, CMB, and BAO data sets, is: $w_0^{\Lambda} = -1.13^{+0.15+0.21}_{-0.13-0.19}$ and $w_{\mathrm{a}}^{\Lambda} = 0.73^{+0.53+0.67}_{-0.69-0.82}$, with $\Omega_{\mathrm{M}} = 1 - \Omega_{\Lambda} = 0.282^{+0.018+0.021}_{-0.017-0.020}$, where each first set of uncertainties includes statistical errors only, and with each latter set including statistical plus systematic errors.

Using these best-fit DE EoS values of $(\Omega_{\Lambda}, w_0^{\Lambda}, w_{\mathrm{a}}^{\Lambda}) = (0.718, -1.13, 0.73)$ in Equation 41, one obtains $j_0^{\mathrm{Obs}} \simeq 2.26$, if one simply plugs those values into this formula for the jerk

parameter without regard to the probability distributions of those parameters. This result for $j_0^{\rm Obs}$ is fairly similar to the value of 2.49 obtained earlier from our $N = 4$ polynomial fit to this Union SNe data alone, it is demonstrably above the flat ΛCDM case, and is not all that far out of the lower end of the range of $j_0^{\rm Obs}$ values quoted previously for our best simulation runs.

Using the uncertainties for each of the parameters given above (averaging the $\pm\sigma$ values in each case), in conjunction with Equation 41, one could naively calculate 1σ error bars for $j_0^{\rm Obs}$ of $\Delta j_0^{\rm Obs} \simeq 0.87$ (stat errors only), and $\Delta j_0^{\rm Obs} \simeq 1.14$ (stat plus syst), actually placing ΛCDM more than 1σ away in both cases. But calculating a valid uncertainty on $j_0^{\rm Obs}$ is not so simple, however: judging by Figure 16 of Kowalski et al. (2008), the uncertainties on w_0^Λ and $w_{\rm a}^\Lambda$ are clearly not independent; and lacking covariance information, we cannot calculate a precise value of $\Delta j_0^{\rm Obs}$.

For the Union2 compilation, however, we have both more recent and more complete information (D. Rubin, 2010, private communication). For their stat-errors-only analysis, they obtain best-fit values of: $(\Omega_\Lambda, w_0^\Lambda, w_{\rm a}^\Lambda) = (0.727, -1.046, 0.160)$, with a (symmetric) covariance matrix of $(c_{\Lambda\Lambda}, c_{00}, c_{\rm aa}, c_{\Lambda 0}, c_{\Lambda {\rm a}}, c_{0{\rm a}}) = (1.782 \times 10^{-4}, 1.305 \times 10^{-2}, 3.053 \times 10^{-1}, 1.766 \times 10^{-4}, 7.254 \times 10^{-5}, -5.672 \times 10^{-2})$; while their stat+syst analysis yields: $(\Omega_\Lambda, w_0^\Lambda, w_{\rm a}^\Lambda) = (0.723, -1.134, 0.585)$, with a covariance matrix of $(c_{\Lambda\Lambda}, c_{00}, c_{\rm aa}, c_{\Lambda 0}, c_{\Lambda {\rm a}}, c_{0{\rm a}}) = (2.776 \times 10^{-4}, 2.794 \times 10^{-2}, 3.370 \times 10^{-1}, -4.747 \times 10^{-4}, 5.658 \times 10^{-4}, -8.803 \times 10^{-2})$. A straightforward calculation (though ignoring the non-Gaussian nature of the parameter distributions, particularly for $w_{\rm a}^\Lambda$) produces final results of: $j_0^{\rm Obs} \pm \Delta j_0^{\rm Obs} \simeq 1.331 \pm 0.986$ (stat errors only), and $\simeq 2.129 \pm 1.292$ (stat+syst). Clearly, the uncertainties on the jerk parameter remain quite large (with ΛCDM now back within 1σ), even when using the Union2 SNe compilation data (without outliers) in conjunction with the CMB and BAO data sets. Furthermore, these two results differ from each other greatly – not only in the size of the error bars for $j_0^{\rm Obs}$, but also in its best-fit central value – with the only difference between them being whether or not one includes systematic uncertainties in the fitting process.

Summarizing the numerical estimates from this subsection: different analyses of the original SCP Union data set yield a variety of values for the jerk parameter, including $j_0^{\rm Obs} \simeq 1.32$ ($N = 3$ case) and $\simeq 2.49$ ($N = 4$ case) for polynomial fits to $d_{\rm L}(z)$ for the SNe alone; and $j_0^{\rm Obs} \simeq 2.26$ is obtained from a non-cosmographic analysis of an evolving-DE EoS with (SN+CMB+BAO) data. For the SCP Union2 data set, alternatively, such polynomial fits – specifically, the $N = 3$ and $N = 4$ fits to $d_{\rm L}(z)$, and the $N = 3$ fit to $\ln[d_{\rm P}(y)/y]$, respectively – yield three different best-values for the jerk parameter of: $j_0^{\rm Obs} \simeq (0.991, 1.050, 1.338)$ for the official 557 SNe Union2 compilation with outlier rejection, and $j_0^{\rm Obs} \simeq (1.151, 2.633, 2.422)$ for the 569 SNe Union2 data set with the ΛCDM-outliers

added back in. Finally, an evolving-DE EoS analysis of Union2 (SN+CMB+BAO) data (557 SNe) yields $j_0^{\mathrm{Obs}} \pm \Delta j_0^{\mathrm{Obs}} \simeq 1.331 \pm 0.986$ (stat errors only), and $\simeq 2.129 \pm 1.292$ (stat+syst).

The conclusions that we may draw from these results can therefore be summed up as follows: (1) It seems likely that the jerk parameter lies within a (very approximate) range of, let us say, $j_0^{\mathrm{Obs}} \sim 0.35 - 3.4$; (2) Very large uncertainties remain, with estimates of j_0^{Obs} remaining highly unstable and sensitive to which fitting functions and variables are used to obtain them (with many cosmological parameter fits having to be discarded on a purely ad-hoc basis), and are also very sensitive to whether or not systematic uncertainties are included in the analysis; (3) Various estimates of j_0^{Obs} are systematically high (somewhat above unity), as is favored by our formalism; though the Cosmological Constant requirement of $j_0 = 1$ is within 1σ of essentially all reasonable fits for the cosmological parameters, and going from the Union to the Union2 SNe compilation does appear to move the results somewhat more towards ΛCDM; (4) The range of jerk parameter values obtained from the $\sim$dozen 'best' simulation runs with our formalism, $j_0^{\mathrm{Obs}} \sim 2.5 - 5.5$ (and $j_0^{\mathrm{Obs}} \sim 2.6 - 3.8$ for the 6 very best runs) is somewhat high compared to this most likely range of j_0^{Obs} as determined from current observations; but the large scatter and uncertainties in the observational data, and the major theoretical simplifications in our calculations (e.g., the lack of recursive nonlinearities in our current models) may plausibly account for this. With further development of our models, and with enlarged and improved SNe data sets, it should be possible in the not-too-distant future to use observations of the cosmic jerk parameter to definitively support or rule out our causally-propagating perturbation formalism as a competitor to Concordance ΛCDM.

7. THE COSMIC FATE: EXPANSION VERSUS DISORDER

One of the signature issues to be addressed by any cosmological paradigm is the question of its predictions for the cosmic future: What is the ultimate fate of the universe? While the toy model presented in this paper for calculating the effects of causal backreaction is too simplified to produce a detailed quantitative answer to this question, it is useful enough for discussing the competing physical processes that are involved in determining the final cosmic fate.

For standard FRW cosmologies – without any Dark Energy or Acceleration – the issue of the ultimate cosmic fate is a theoretically simple one, depending only upon the total mass-energy density, Ω_{Tot} (e.g., Kolb & Turner 1990). A closed universe ($\Omega_{\mathrm{Tot}} > 1$) will recollapse in an 'infinitely' dense "Big Crunch"; whereas a universe with insufficient closure density will either expand eternally (open universe, $\Omega_{\mathrm{Tot}} < 1$) or asymptotically approach zero expansion speed (critical/flat universe, $\Omega_{\mathrm{Tot}} = 1$), in either case approaching a thermodynamic 'heat

death', with an absolute and final cooling of the universe into a "Big Freeze" (or "Big Chill"). Cosmic acceleration complicates the issue, making it dependent upon the nature of the Dark Energy. If the Dark Energy should be some sort of Quintessence that ultimately loses its negative-pressure properties and turns into a form of 'normal' matter, then the cosmic fate would revert to the standard case of depending solely upon Ω_{Tot}. On the other hand, if the Dark Energy retains its accelerative potency – such as in the case of the Cosmological Constant, Λ – then the acceleration would never stop, and all matter not tied to the final large cluster containing all bound mass in our region of space ("Milkomeda", e.g., Cox & Loeb 2008) would fly away out of causal contact; the total amount of information (and thus useful energy for doing work) available to any observer would therefore be finite even for an eternal universe, eventually causing the ability to do computations (and thus the possibility of sustaining life) to ultimately fade away (Krauss & Starkman 2000). Or even more severely, if the Dark Energy should actually increase in potency over time – such as would be the case for Phantom Energy ($w^{\Lambda} < -1$) – then the universe could end in a "Big Rip", leading to a Cosmic Doomsday where all cosmic structures, objects, and even atoms are ultimately ripped apart (Caldwell et al. 2003). Other exotic models naturally lead to other possibilities, such as a cyclic universe with a "Big Bounce" (e.g., Veneziano 2004), and so on.

For the causal backreaction formalism that we present here, in which exotic contributors such as Dark Energy, Higher Dimensions, etc., are unnecessary, the situation is less complex in theory while being more complicated in practice. While there is no more need to understand the behavior of any independent substance or modified physics that exerts some 'external' control over the rest of the universe, there is now the necessity of calculating a fully self-consistent solution for the evolution of an increasingly inhomogeneous cosmos, whose own perturbations are causing itself to accelerate (or at least to appear to do so). A real solution of this type would require a method of dealing with gravitational nonlinearities, along with fully three-dimensional modeling to deal with the ultimate breakdown of our smoothly-inhomogeneous approximation. But even without such tools at our disposal here we can at least delineate some of the more likely possibilities, to see how they compare to the popularly regarded cosmic fates mentioned above, and also to see if any new wrinkle appears that modifies these possibilities in some interesting way.

In terms of our clumping-perturbed metric, Equation 13, the effects of causal backreaction completely take over the cosmic evolution as $I(t)$ approaches unity; and the model breaks down for $I(t) \approx 1$ (and is completely meaningless for $I(t) > 1$), as gravitational nonlinearities cause the failure of our approximation of linearly summing together the individual Newtonian gravitational potentials from different virialized mass clusters. Two questions naturally arise: (1) Will this $I(t)$ function ever actually approach unity? (2) And if so, what really happens to the cosmic expansion as a result?

Considering the latter question first, to see what the stakes are, it seems likely that 'real' acceleration due to backreaction, in the sense of q_2 from Section 2 above, would finally become cosmologically important. In other words, the fitting problem of finding an averaged cosmology to represent a very inhomogeneous universe would become a dominant concern, and accelerated volume expansion due to the continuing causal backreaction, now modified by truly nonlinear gravitational effects, would fully control the evolution of the universe. In that case, we suppose that it is possible for the result to be a runaway acceleration, as potent perhaps as that from a Cosmological Constant (or maybe even as strong as a form of Phantom Energy), since the effective value of w^{Obs} would be due to gravitational energy rather than due to the exotic nature of any particular physical substance, and hence not be subject to the bounds (i.e., $w \geq -1$) normally imposed upon perfect fluids by the dominant energy condition. (One could even speculate that such a process may have had relevance to the pre-inflation very early universe – given its presumably chaotic and inhomogeneous nature – perhaps accounting for some or all of the acceleration usually credited to the Inflaton field(s); assuming, of course, that such a process could be shown to be theoretically possible given causality restrictions.)

Alternatively, some authors have argued that backreaction due to matter inhomogeneities cannot be responsible for Λ- or Phantom-like acceleration; or that if they can be, temporarily, then such an acceleration cannot continue eternally. Kasai et al. (2006) use the volumetric conservation of pressureless dust to make arguments towards a no-go theorem for acceleration from nonlinear backreaction; but their calculations are based upon the same cosmologically inapplicable approximations discussed previously in Section 2. Also, they use the same function $a(t)$ as both their unperturbed scale factor, and as their measure of cosmic volume (i.e., $a \propto V^{1/3}$) in the perturbed metric – two roles for $a(t)$ that are not simultaneously valid in a model such as ours, for example, since $I(t), \dot{I}(t) \neq 0$ alters the evolving physical volume of a perturbed region away from a simple proportionality to a^3 (e.g., recall Section 5.2).

Bose & Majumdar (2011), on the other hand, argue that whatever specific physical process may actually be generating the current cosmic acceleration, that it cannot continue eternally, due to backreaction from the *future* cosmological event horizon which would exist in a universe that accelerated forever. Despite the use of the Buchert formalism (e.g., Buchert & Ehlers 1997) – along with its unacceptable assumptions for causal backreaction (irrotational dust, zero Newtonian backreaction, etc.) – in their calculations of the backreaction from this future horizon, their results are obtained without necessarily presuming Buchert-generated acceleration; and they claim that the acceleration must eventually stop regardless of the mechanism behind it. Looking closely at their results, however, one sees that this claim only really works when one considers a large region made up of subregions

(two, in their calculations) that are each individually decelerating; and in that case, all they have proven is that the Buchert-generated acceleration of the combined region eventually becomes weaker than the total sum of the individual decelerations of the component subregions. If one of the subregions is actually accelerating on its own through some non-Buchert mechanism (such as via our causal backreaction method, nonzero Λ, etc.), and possesses a sufficiently large initial fraction of the total volume – or if *all* subregions are individually accelerating ($\alpha > 1$, $\beta > 1$ in their terminology) – then inspection of their Equations 4 and 9 shows that the overall acceleration $\ddot{a}_D$ *cannot* go to zero for positive, finite t, unless the 'apparent volume fraction' of a component subregion as part of the total were to become negative, which is obviously not consistent.

While the conclusions that one may draw from calculations such as these are debatable, and the ideas behind them are still in too early a stage of development to be definitive, they still make the important point that there may be 'global' considerations that either prevent or ultimately shut down a long-term cosmic acceleration due to effects that go beyond those contained within any simplified model like our formalism. It is therefore important to keep such caveats in mind when considering the so-called 'ultimate' possible fates of the universe, below; but regardless of the *eternal* validity of any conclusions, at least we can use our model to elucidate some reasonably long-term effects of the crucial phase transition which must undoubtedly occur when (and if) the causal backreaction becomes nonlinear in strength.

Now considering our former question – will $I(t)$ really approach unity? And even more seriously, will gravitationally nonlinear effects due to causal backreaction come to dominate (and in some sense 'permanently' control) the evolution of the Universe? In our simulation runs, $I(t)$ increases monotonically, and gets quite close to 1 for the strongest-'accelerating' cosmological models; it would therefore be useful to estimate how large $I(t)$ can get within our formalism.

In the essential physics of backreaction with causal updating, there are several different processes competing against one another – some enhancing the apparent (or ultimately real) acceleration, and some damping it – and which way the balance tilts between these factors will largely (but as we will see in the end, not completely) determine the ultimate cosmic fate. The most obvious factor enhancing the acceleration is the increasing 'clumpiness' of the universe, represented in this paper by clumping evolution function $\Psi(t)$, which increases monotonically over time. In our formalism, however, the extent of clumping must saturate as $\Psi(t) \to 1$, thus limiting its ability to continue driving the growth of $I(t)$; and this seems like a physically realistic constraint on $\Psi(t)$, representing how virtually all cosmic matter should eventually reach a 'maximally clumped' state, in which the universe consists almost entirely of steadily expanding voids, punctuated only by a sparse scattering of fixed-size

massive clumps that no longer expand with the universe, but which manage to virially support themselves against singular collapse.

But even more important than the increasing clumpiness of any given region of space, is how the *information* about distant clumpiness (i.e., $\Psi[t_{\text{ret}}(t, \alpha)]$) can get to an observer from farther and farther distances with increasing t, as the volume of space with significant clumping within the observer's causal horizon ($\alpha \leq \alpha_{\text{max}}$, cf. Equation 10) grows rapidly with time. This effect is counterbalanced, however, by the continuing expansion of the universe, which dilutes such backreaction by taking any given mass 'clump' farther away from the observer as t increases (specifically referring to the factor of $1/[a_{\text{MD}}(t)]$ in $R_{\text{Sch}}(t)$; cf. Equations 8,11). These opposing factors are represented by α_{max} and $(t_0/t)^{2/3}$, respectively, in Equation 12 for $I(t)$. Ignoring the time dependence in $\Psi[t_{\text{ret}}(t, \alpha)]$ for the moment, one approximately gets $I(t) \propto [(t_0/t)^{2/3} \, \alpha_{\text{max}}^2]$; but since $\alpha_{\text{max}} \simeq (t/t_0)^{1/3}$ for $t_0 \gg t_{\text{init}}$ (cf. Equation 10), we see that the overall time dependence ultimately cancels out of $I(t)$ at sufficiently large t. We can interpret this to mean that the increasing backreaction effect due to the expanding causal horizon *also* saturates, due to competition against the cosmic expansion itself, with $I(t)$ approaching a constant value at times very late compared to t_{init} (and very late also compared to the time by which the clumping evolution, $\Psi(t)$, has mostly saturated near unity). What we need to know, of course, is *how large* this asymptotically constant value of $I(t)$ will be; an obviously model-dependent question.

Treating $\Psi[t_{\text{ret}}(t, \alpha)]$ as a step function – equal to $\simeq 1$ for $\alpha \leq \alpha_{\text{max}}$ (with $t_{\text{init}} \simeq 0$), and zero beyond there – one gets $I(t_0) \simeq 6 \gg 1$, a result deep into the regime requiring a gravitationally nonlinear treatment; but this of course is an unrealistic upper bound on the effects of clumping, representing a universe that became fully clumped almost instantaneously, saturating at very early cosmic times (i.e., $\Psi(t) \to 1$ at $(t/t_0) \ll 1$). Trying more realistic (and continuous) functional forms for $\Psi(t)$, with a fairly recent saturation of clumping – in particular, considering the three functions used for our clumping evolution models, as defined above in Equations 22a-c – the maximum possible value of $I(t)$ for each clumping model can be obtained for $t_{\text{init}} \simeq 0$, $t \to t_0$, and $\Psi(t_0) \equiv 1$. Doing this, it is easy to show that $I_{\text{max}}(\Psi_{\text{Sqr}}) = 3/14$, $I_{\text{max}}(\Psi_{\text{Lin}}) = 3/5$, and $I_{\text{max}}(\Psi_{\text{MD}}) = 1$. Thus the 'strongest' of our clumping evolution models (Ψ_{MD}) can definitely produce results strong enough to violate the approximation of gravitational linearity; and even stronger models, with more gradual clumping growth (that is, an earlier start to significantly large Ψ) – given as $\Psi(t) \propto (t/t_0)^N$ with $N < 2/3$ – would have I_{max} values that easily exceed unity (e.g., $I_{\text{max}} = 2$ for $N = 1/3$, $I_{\text{max}} = 6$ for $N = 0$, etc.). In short, causal backreaction as prescribed by our formalism can clearly be strong enough to not only induce an apparent acceleration, but also to break down our linearized gravitational approximation entirely, indicating a possible runaway acceleration for the real universe, itself. But whether that possibility is realized, or whether

the strongest effects of causal backreaction fall short of it, depends extremely sensitively upon the precise time-dependence of the evolution of mass clumping and virialization in the universe.

One important moderating factor for backreaction has been completely left out of these estimations, however: the recursive nonlinearities discussed previously, representing how the metric perturbations due to early inhomogeneities will slow down the causal updating that brings in new inhomogeneity information. In addition, some extra volume expansion is created (cf. Equation 8 and subsequent discussion) that adds to the ordinary FRW Hubble expansion in pulling clumped masses farther apart from one another, thus diluting these backreaction effects to a degree greater than that predicted by Equation 12 and numerically estimated above. These effects should not only reduce the value of j_0^{Obs} predicted by our simulations (as mentioned in Section 6.2.2), but may in a larger sense serve as a causal backreaction 'regulator', feeding back upon that very backreaction to slow down its effects upon the universe whenever the acceleration caused by it begins going into a runaway mode. But such a picture of self-regulating cosmic acceleration remains purely speculative until a more complete formalism and simulation program exist which can quantitatively model the effects of these recursive nonlinearities into the far future.

Thus the variety of 'ultimate' cosmic fates potentially emerging from our causal backreaction paradigm seem to fall into a few fairly recognizable categories, depending in detail upon the evolution of the growth rate of self-stabilized cosmic structures. On the one hand, the effects of causal backreaction may weaken or even saturate, leading either to a continuing low level of 'acceleration', or to the kind of coasting/mildly slowing behavior of open/flat universes – in either case resulting in a similar kind of Big Freeze as would have occurred in the normal ($\Omega_{\text{Tot}} \leq 1$) FRW case. Alternatively, if the cosmic structure formation evolution leads to a stronger, self-sustaining level of backreaction, then it is conceivable that the fading-away behavior (the "Big Fade"?) of the rest of the universe as would occur for an exponentially-expanding Cosmological Constant universe may not be a bad approximation. Or in extreme cases (probably less likely), some form of Big Rip could conceivably occur. Qualitatively speaking though, the most reasonable scenario in our paradigm may be some kind of stop-and-go accelerating behavior, due to the self-regulating nature of causal backreaction with recursive nonlinearities. This could lead to a situation where the universe is always 'riding the edge' of a runaway acceleration, but never quite getting there – perhaps in a permanent state of acceleration that oscillates over time but stays relatively close to the level of acceleration that we see today, with the same relationship that we currently see between effective FLRW observables (i.e., $\Omega_\Lambda^{\text{Obs}} \sim \Omega_{\text{M}}^{\text{Obs}}$ to within an order of magnitude), with these conditions remaining generically true for the foreseeable cosmological future.

Now, the variety of possible cosmic fates considered here is of course not unique to our formalism – each of these possibilities can easily be reproduced by a version of Dark Energy with some appropriately evolving equation of state. The difference is that instead of being dependent upon the unknown (and perhaps ultimately unknowable, in fine detail) theoretical nature of a master Dark Energy field running the rest of the universe, the cosmic fate here is due to the normal gravitational forces and motions of matter, which – though very complicated in practice – will depend only upon calculations using known laws of physics (once the nature of the Dark Matter is known!), and the long-explored mechanisms of general relativity.

And yet, one more wild card still remains, one more consideration regarding the plausibility of these possible cosmic fates, which will tell us which one really is the most probable description of the cosmic future; and the answer is: *none of them!* In the entire discussion of our formalism in this paper, there is one 'elephant in the room' that has not been analyzed, because it cannot be modeled in this way – and that is the breakdown of our smoothly-inhomogeneous approximation itself. As the universe grows older, and ever-larger bound structures have time to form, the universe will eventually become inhomogeneous and anisotropic on a truly cosmological scale. Thus the Cosmological Principle will break down entirely as a usable approximation, and vast cosmic structures will eventually exert powerful tidal forces that are strong enough even to threaten bound galactic clusters. While this is also not a theoretical phenomenon unique to our specific formalism, once again there is a key difference: whereas typically one expects the dominant perturbing influences on local structures to be the closer ones (cosmologically speaking), consisting of just one or a few definable, huge objects, our causal updating analysis shows that the acceleration effects due to backreaction are dominated by the *farthest* distance out to which one can causally see clumped structure. Thus in the distant future, the gravitationally perturbative effects acting upon otherwise 'local' structures will be dominated by a multitude of different inhomogeneities, located in all directions, that are cosmologically-extreme distances away, and that together represent an almost unimaginable amount of mass in clumped structures. Such tidal or shearing forces will continue to grow and grow – as long as the causal flow of information can continue to come in from ever-more-distant parts of the universe – until gravitationally bound local structures are chaotically ripped apart. In far more anisotropic fashion than the Big Rip of Phantom Energy – if perhaps less destructively, since *some* form of recursive nonlinearities and dilution will eventually slow the process – this evolution will herald the end of the smooth cosmic expansion existing at present, rendering all FRW/FLRW models obsolete. The first hints of such behavior in our universe may already be making themselves evident, perhaps accounting for the observed phenomenon known as the "Dark Flow" (Kashlinsky et al. 2008, 2010) – or at least accounting for the causal part

of it, if the effect indeed extends to 'superhorizon' scales. One thing is clear, according to our causal backreaction formalism: this Dark Flow (assuming it to be observationally real and at least partly causally generated) is just the beginning of a greater trend of inhomogeneity and anisotropy that will be acting upon all local cosmic structures, and doing so much more rapidly, violently, and chaotically than would be expected in more conventional (i.e., non-causal, Poisson-Equation-based) models of structure formation.

The *real* fate of the universe will therefore not be a Big Crunch, a Big Freeze, a Big Fade, a Big Rip, or even a Big Bounce – instead it will end (if one can call it an 'end') in something entirely more appropriate to the entropy-riddled universe that we all have known: a Big Mess.

8.   SUMMARY AND CONCLUSIONS

We may sum up the motivations, methods, and results of this paper as follows. Our starting point is the search for an alternative to Dark Energy as an ingredient in the nascent Cosmic Concordance, without resorting to the increased theoretical complexity represented by modifications to gravity, or the non-Copernican 'specialness' implied by a local cosmic void. The problems with Dark Energy are well known, including the coincidence and magnitude fine-tuning problems for a pure-Λ Cosmological Constant, and the stability problem for a more dynamical form of Dark Energy possessing the ability to cluster spatially. The latter (DDE) case requires the ad-hoc addition of some form of new pressure term to support it against collapse, given the *locally attractive* nature of the negative pressure required by the DDE to power the cosmic acceleration. Such a new pressure term, if adiabatic (e.g., degeneracy pressure), would represent a form of *positive* pressure contributing to a cosmic *deceleration* that partially or totally nullifies the acceleration meant to come from the DDE; and if non-adiabatic (due to some new effects from the DDE Lagrangian), would invalidate the Dark Energy as a perfect fluid, thus calling the usual (accelerative) cosmic implications of $w^{\mathrm{DE}} < 0$ in the FLRW acceleration equation into question.

Emphasizing the importance of the fact that 'negative pressure' is locally attractive in character – rather than repulsive, as it is often regarded and popularly described – and that normal gravitational attraction therefore represents (at least in a non-technical sense) a form of negative pressure, we have therefore made a virtue of necessity by recruiting cosmological structure formation itself, based upon ordinary gravitational forces, as the driver of the observed (possibly apparent) acceleration. This approach, known generally in the literature as "backreaction", removes all of the aforementioned problems by both eliminating the need for any Dark Energy species, and by solving the Coincidence Problem by naturally linking

the onset of cosmic acceleration with the emergence of cosmic structure, which inevitably leads to the creation of observers such as ourselves just in time to see this 'coincidence'.

Noting that several versions of backreaction have been largely unsuccessful in their attempts to account for the observed acceleration, and that powerful arguments have been advanced which claim that backreaction as a paradigm *cannot* be made strong enough to succeed at this task, we have described how each of these arguments (and the backreaction mechanisms which they address) are functionally invalid. This lack of validity is not generally due to mathematical flaws within the arguments themselves, but due to their inapplicability to the real universe: all such "no-go" arguments or theorems are based upon one or more essential simplifications so restrictive that they eliminate from consideration all of the actual physics that is responsible for the crucial 'accelerative' effects.

Different arguments against backreaction fail for different reasons. No-go conclusions drawn from the Raychaudhuri equation are invalid because they neglect vorticity (or more precisely, the *square* of the vorticity, ω^2), thus ironically dropping the dominant physical force in the universe which prevents the collapse of all (non-solid) structures into singularities, and which is a positive semi-definite quantity that does not go to zero regardless of how large a scale one uses for averaging it. Swiss-Cheese models (typically using Schwarzschild or LTB metrics for the holes) underestimate backreaction effects not only because they lack a mechanism for modeling vorticity and virialization, but more crucially because they partition the cluster-forming universe into a discrete set of non-intersecting, non-interacting volumes – a construct completely at odds with how structure formation works in the real universe, where overdensities extend their reach far beyond their local domains to cause vast inflows, and where any given region of space feels the gravitational effects of many such influences *combined together*, rather than being impacted by one influence while being hermetically sealed off from all of the others. Lastly, there are no-go arguments based upon the generally Newtonian nature of metric perturbations and mass flows, apparently demonstrating that the sum total effects of backreaction must be small (or according to the formalism of Buchert and collaborators, identically zero); but such arguments are flawed due to the non-causal basis of the calculations (since they drop all signal travel-time delays, gravitomagnetic effects, and time derivatives of the perturbation potential), and because they depend upon 'smallness' arguments that are irrelevant in the face of the combined perturbative effects of innumerable cluster-forming masses acting upon every single region of space within their causal grasp.

The true utility of such no-go arguments, however, is that gaining an understanding of why and where they fail to apply can guide us in the construction of a more successful, working form of backreaction. The central challenge is figuring out how best to model the crucial cosmic phase transition from smooth matter distributed largely homogeneously

throughout space, to most matter being concentrated into a fairly randomly-distributed collection of vorticity-stabilized, self-virialized, clumped structures. In this paper, we model the situation heuristically and simply by representing the dominant perturbation due to each 'clump' as being (the Newtonian tail of) a Schwarzschild metric embedded in the expanding universe, superposed on top of the background FRW metric – where we utilize the fact that the vorticity (or velocity dispersion) stabilizing each structure will add its own positive contribution to the volume expansion, as per the Raychaudhuri equation. We then combine the perturbative contributions of all such objects by summing them and averaging over location and direction, producing a "smoothly-inhomogeneous" universe model that neither possesses nor requires a local void or spatial variations. Instead, it possesses only a single clumping evolution function that increases over time, representing the ever-growing fraction of cosmic matter located in self-stabilized clusters, rather than remaining in the smooth background. Lastly, and perhaps most importantly, we introduce the mechanism of "causal updating" in order to represent the crucial process of how the information about this growth of clustering propagates through the universe to any given observer, coming in from the very limits of the causal horizon out to which that observer has had time to see clumped structure.

Employing these physical considerations, we have constructed a formalism of "causal backreaction" for calculating how the smoothly-inhomogeneous metric evolves over time, given any pre-specified clumping evolution function $\Psi(t)$. From that metric solution, we can calculate a luminosity distance curve as a function of observed redshift z^{Obs}, for comparison to residual Hubble curves measured via standard candles (particularly Type Ia supernovae), in order to check if an observed (possibly apparent) acceleration of the right magnitude and z^{Obs}-dependence has been observed. Additionally, we can use our formalism to calculate several key cosmological observables produced by that function $\Psi(t)$ – such as t_0^{Obs} (age of the universe), $\Omega_{\mathrm{M}}^{\mathrm{FRW}}$ (matter density as a fraction of the unperturbed FRW critical density), w_0^{Obs} (apparent Dark Energy equation of state), j_0^{Obs} (cosmic jerk parameter), and $l_{\mathrm{A}}^{\mathrm{Obs}}$ (characteristic angular scale of the CMB acoustic peaks) – testing whether the essential results of an alternative cosmic concordance have been achieved for causal backreaction operating upon such a clumping evolution function. Altogether, this adds up to a sterner test than the mere production of an apparent acceleration.

Armed with this formalism, and using a set of 60 clumping evolution functions designed for simplicity and using model input parameters determined from straightforward astrophysical considerations, we have found a ∼dozen-plus solutions that fit the SCP Union Compilation SNe data essentially as well as the best-fit ΛCDM model. Furthermore, about half of these models give good values (within reasonable theoretical and observational uncertainties) for all other calculated cosmological parameters. That is, these few 'best' models (obtained

without even conducting a full optimization search over the model parameter space) have successfully produced an alternative concordance, at least to the extent of solving the cosmic Age Problem, achieving spatial flatness (for the unperturbed, pre-clumping FRW universe), and being reasonably consistent with the CMB peak positions; and they achieve all of these goals – along with explaining the observed acceleration – in a matter-only universe, without any form of Dark Energy component.

Now given that these important objectives – particularly the difficult one of achieving flatness by bringing the total cosmic density from $\Omega_0 \sim 0.3$ up to $\Omega_0 \simeq 1$ – have been major pillars in the argument for Dark Energy, our ability to achieve them with matter alone, simply by including the ordinary gravitational effects of causal backreaction, would seem to turn all forms of Dark Energy (Λ, Quintessence, etc.) into excess theoretical baggage. Nevertheless, since one cannot *prove* a theory with Occam's Razor alone, we have sought in this paper to find ways to distinguish our causal backreaction paradigm from any kind of Dark Energy (where possible), or at the very least to distinguish it from the simplest form of Dark Energy – i.e., a Cosmological Constant. One generic implication of using backreaction to generate the cosmic acceleration is that this places an increased emphasis upon the possible existence of larger-scale anisotropies and inhomogeneities than those expected from the standard perturbed-FLRW cosmologies. To that end, we have commented above about potential observational signs of such deviations from the Cosmological Principle that have been seen over the past few years by other researchers, as well as referring to work by this author and others using Type Ia SNe data sets to search for direct signals of possible large-scale anisotropies.

Additionally, there are potential discrepancies for ΛCDM in particular, including the cuspy CDM halo problem and the possible dearth of dwarf satellite galaxies, which might be naturally explainable due to clustering-induced feedback occurring in the causal backreaction paradigm. Finally, there would of course be alterations to virtually all other cosmological observables due to causal backreaction, providing any number of possible signals in the complementary cosmic data sets that can be used for testing our paradigm; though detailed calculations of most of those alterations are beyond the scope of this current paper. One altered parameter that we have in considered in detail here, however, is the jerk parameter j_0^{Obs}, which for flat ΛCDM is required to be exactly unity. For our cosmologically successful simulation runs using the causal backreaction model, on the other hand, $j_0^{\mathrm{Obs}} > 1$ is clearly preferred, a general result which we have seen is supported reasonably well (though statistically very weakly) by trends in current cosmological data. Quantitatively speaking, the best runs in this paper predict an approximate range of $j_0^{\mathrm{Obs}} \sim 2.5 - 5.5$ (with a more narrow range of $j_0^{\mathrm{Obs}} \sim 2.6 - 3.8$ for the very best few runs), which is perhaps high compared even to the upper range of observational estimates; but as explained previously, such values com-

puted here from our simulation runs are almost certainly higher than they should be due to current oversimplifications of our model (e.g., the absence of what we have termed "recursive nonlinearities", etc.), which must be remedied with further theoretical development of these calculations.

Having obtained such favorable results for producing an alternative concordance without Dark Energy, we then considered the 'ultimate' cosmic fates possible for a universe dominated by causal backreaction. Though the mathematical toy model introduced in this paper is still too simple at this point to yield detailed quantitative predictions about the universe in the far future, we expect the possibilities to be quite similar to the usual variety of potential fates for a universe dominated by various forms of Dark Energy with its typical designer equations of state. But there are two major differences, however: (1) With the acceleration (apparent or real) being provided by the detailed processes of structure formation in the matter itself, rather than from some 'external' material component with its own independent equation of state evolution, it seems far more likely that a self-limiting, stop-and-go form of acceleration will be the ultimate cosmic fate, rather than any complete runaway scenario (such as the Big Rip of Phantom Energy); and: (2) Given the unwonted importance here of inhomogeneities at great cosmological distances for determining 'local' gravitational behavior, causal backreaction is likely to lead to far more extreme and asymmetrical tidal forces on locally bound objects, eventually leading to a far messier and more chaotic universe than anything expected from 'normal' models of inhomogeneity evolution, given any form of cosmic acceleration this side of a Big Rip.

Expressed in one sentence, if one asks, "What is the force behind the cosmic acceleration?", the answer we would give is that it is not a 'force' at all: rather, the motivating effect may be called "the Schwarz" – consisting of the total, summed effects of the Newtonian tails of Schwarzschild-like metric perturbations, produced by the virialization of innumerable self-stabilized structures filling the universe, with these influences propagating causally towards all observers from the extreme edges of their observable cosmic horizons.

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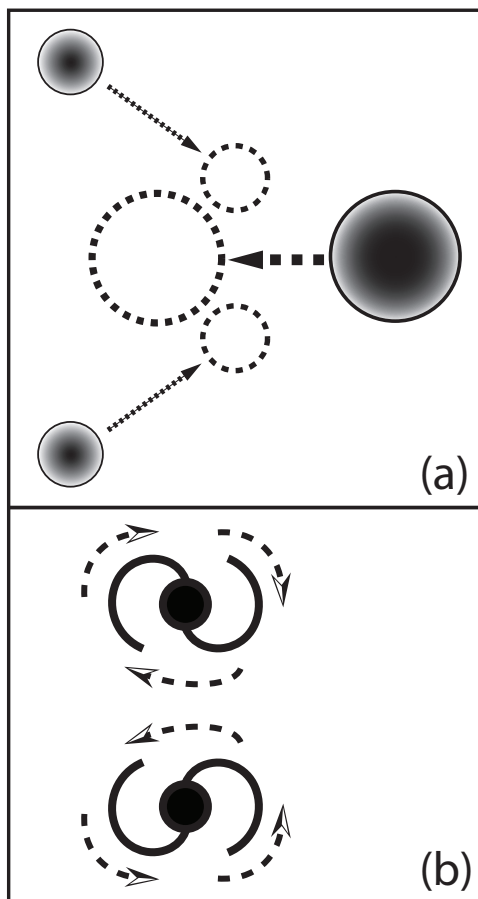
Fig. 1.— Simplified example of a physically permissible (though inelastic and nonadiabatic) merger of three nonrotating objects (a) into two rapidly spinning objects (b). (Not shown: ejected material containing mass and energy but zero total angular momentum.) Here, the angular momentum averaged over the volume of the combined system, $\langle \boldsymbol{L} \rangle$, remains ≈ 0, though $\langle L^2 \rangle$ goes from ≈ 0 to a large value.
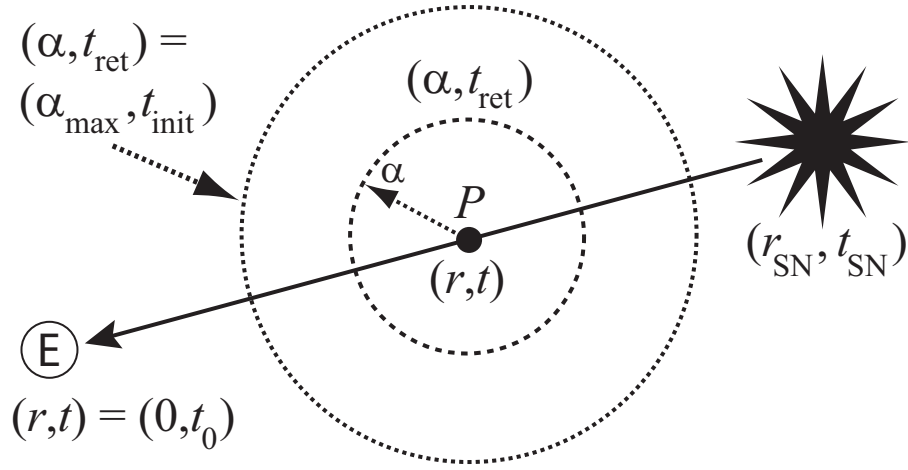
Fig. 2.— Geometry for computing the inhomogeneity-perturbed metric at each point along the integrated path of a light ray from a supernova to our observation point at Earth.
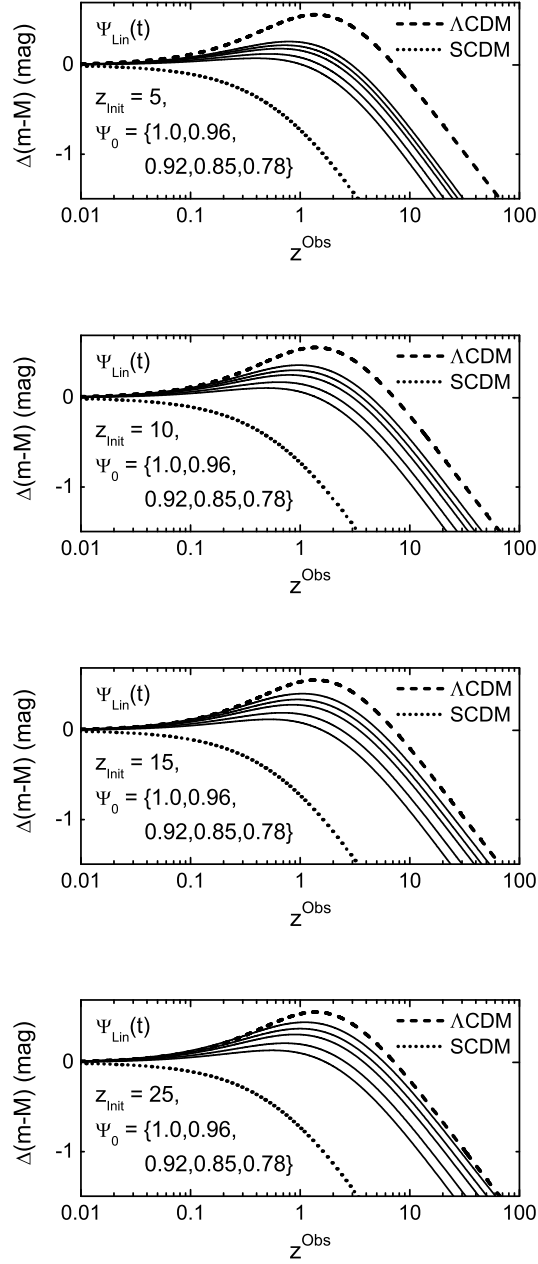
Fig. 3.— Residual Hubble diagrams computed from our apparent acceleration model using $\Psi_{\mathrm{Lin}}(t)$ clumping functions. Different panels represent initial-clumping epochs of (respectively): $z_{\mathrm{init}} = (5, 10, 15, 25)$. Each plot depicts the usual flat ΛCDM ($\Omega_\Lambda = 0.73$) and SCDM cosmologies, shown versus five of our simulation runs using present-day clumping parameters of (plotted highest to lowest): $\Psi_0 = (1.0, 0.96, 0.92, 0.85, 0.78)$.
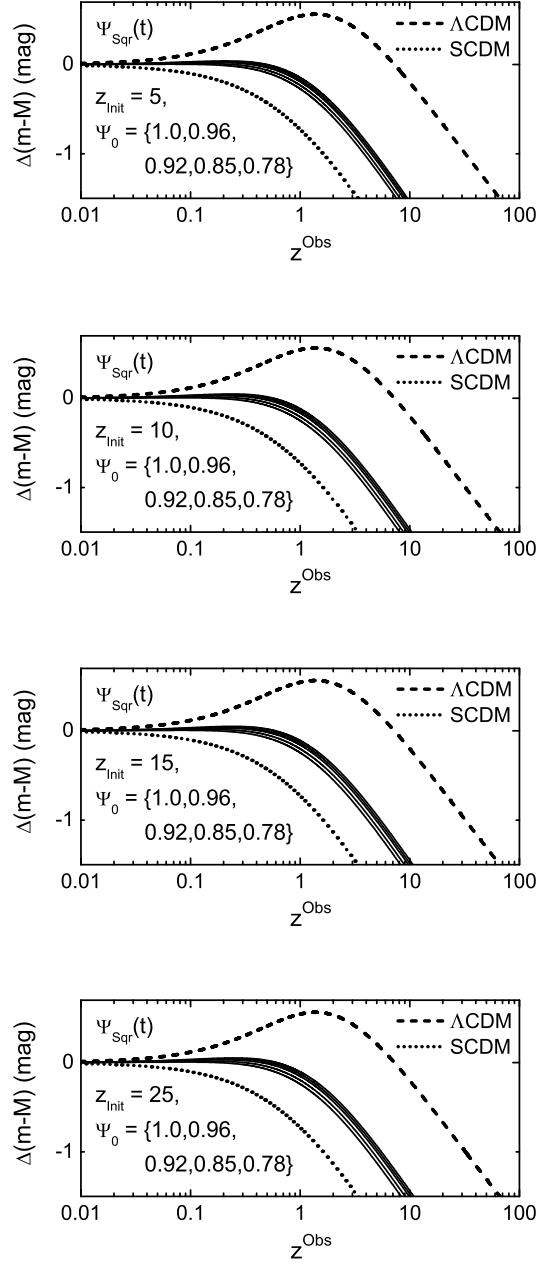
Fig. 4.— Residual Hubble diagrams computed from our apparent acceleration model using $\Psi_{\mathrm{Sqr}}(t)$ clumping functions. Different panels represent initial-clumping epochs of (respectively): $z_{\mathrm{init}} = (5, 10, 15, 25)$. Each plot depicts the usual flat ΛCDM ($\Omega_\Lambda = 0.73$) and SCDM cosmologies, shown versus five of our simulation runs using present-day clumping parameters of (plotted highest to lowest): $\Psi_0 = (1.0, 0.96, 0.92, 0.85, 0.78)$.
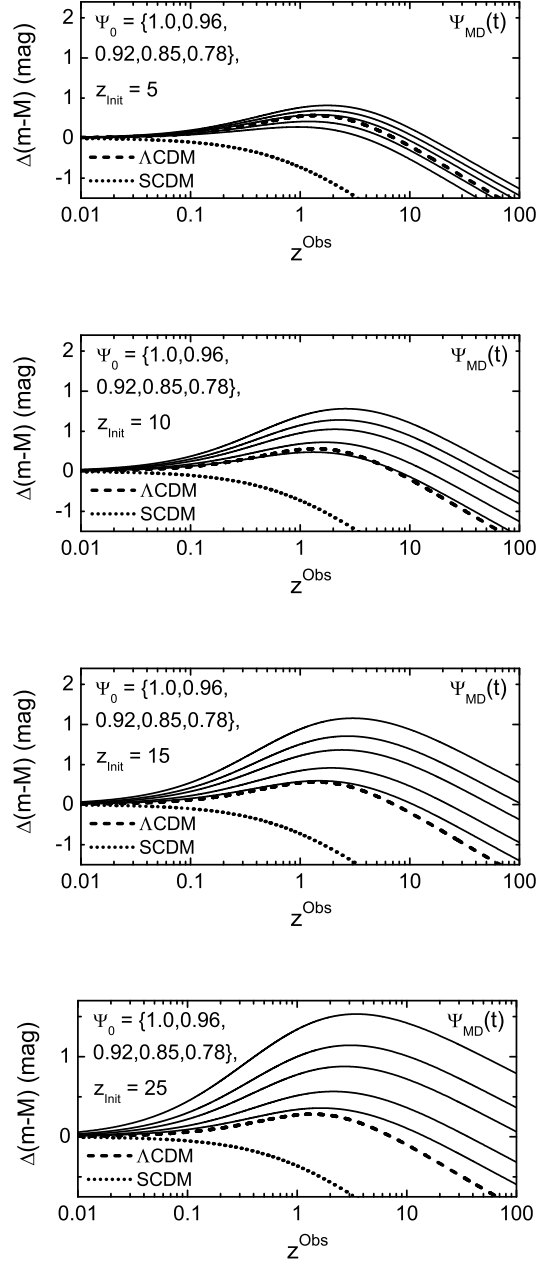
Fig. 5.— Residual Hubble diagrams computed from our apparent acceleration model using $\Psi_{\mathrm{MD}}(t)$ clumping functions. Different panels represent initial-clumping epochs of (respectively): $z_{\mathrm{init}} = (5, 10, 15, 25)$. Each plot depicts the usual flat ΛCDM ($\Omega_\Lambda = 0.73$) and SCDM cosmologies, shown versus five of our simulation runs using present-day clumping parameters of (plotted highest to lowest): $\Psi_0 = (1.0, 0.96, 0.92, 0.85, 0.78)$.
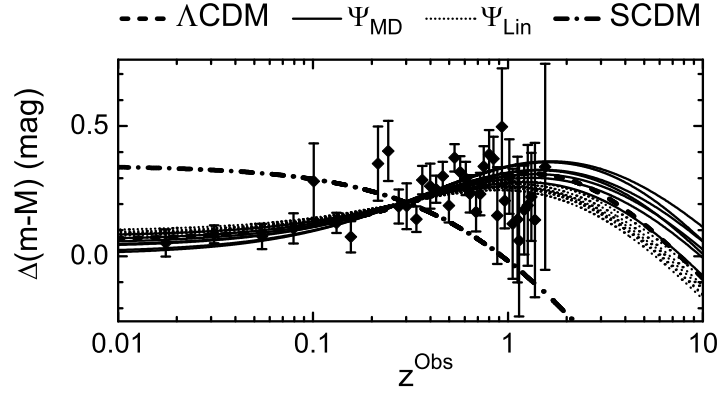
Fig. 6.— Residual Hubble diagrams for the thirteen 'best' simulation runs, as described in the text, of our inhomogeneity-perturbed apparent acceleration model, shown along with the best-fit flat SCDM and Concordance ΛCDM ($\Omega_\Lambda = 0.713$) cosmologies. From highest to lowest (at $z^{\mathrm{Obs}} \simeq 10$), the plotted $\Psi_{\mathrm{MD}}(t)$ curves have the parameters: $(\Psi_0, z_{\mathrm{init}}) = (0.96, 5)$, $(0.92, 5)$, $(0.85, 5)$, $(0.78, 10)$, $(0.78, 15)$, $(0.768, 14)$, $(0.78, 25)$. From highest to lowest (at $z^{\mathrm{Obs}} \simeq 10$), the plotted $\Psi_{\mathrm{Lin}}(t)$ curves have the parameters: $(\Psi_0, z_{\mathrm{init}}) = (1.0, 10)$, $(1.0, 15)$, $(0.96, 15)$, $(1.0, 25)$, $(0.96, 25)$, $(0.92, 25)$. Shown along with these curves are the SCP Union SNe data, here binned and averaged for visual clarity (bin size $\Delta\mathrm{Log}_{10}[1 + z^{\mathrm{Obs}}] = 0.01$). Each theoretical model is individually optimized in H_0^{Obs} to minimize its χ^2_{Fit} for the full SCP Union SNe data set (see Table 1). For simplicity, instead of moving the SNe data up or down for each different optimized H_0^{Obs} value, the optimization is depicted here by plotting the residual Hubble diagram of the SNe data versus a coasting universe of a single, fixed Hubble constant ($H_0^{\mathrm{Obs}} = 72$ km s^{-1}Mpc^{-1}), and then displacing each theoretical curve vertically, relative to the SNe data, as appropriate for each fit.

Table 1: Cosmological Parameters Derived from the 'Best' Runs of our Set of Simulations

$(\Psi_0, z_{\rm init})$	$\chi^2_{\rm Fit}$ [a]	$P_{\rm Fit}$ [b]	I_0 [c]	$z^{\rm Obs}$ [d]	$H_0^{\rm Obs}$ [e]	$H_0^{\rm FRW}$ [f]	$t_0^{\rm Obs}$ [g]	$\Omega_{\rm M}^{\rm FRW}$ [h]	$w_0^{\rm Obs}$	$j_0^{\rm Obs}$	$l_{\rm A}^{\rm Obs}$
$\Psi_{\rm Lin}$ *Clumping Model Runs*											
(1.0,25)	312.1	0.362	0.57	1.12	69.95	40.68	13.56	1.037	-0.817	3.45	284.2
(1.0,15)	313.3	0.344	0.55	1.12	69.64	41.63	13.45	0.971	-0.784	3.11	287.6
(1.0,10)	315.4	0.314	0.52	1.12	69.27	42.80	13.30	0.897	-0.746	2.76	291.3
(0.96,25)	314.8	0.323	0.55	1.11	69.38	41.57	13.38	0.968	-0.759	2.92	285.9
(0.96,15)	316.6	0.297	0.52	1.11	69.11	42.46	13.28	0.911	-0.732	2.67	289.1
(0.92,25)	319.0	0.265	0.53	1.11	68.85	42.44	13.21	0.905	-0.706	2.49	287.6
$\Psi_{\rm MD}$ *Clumping Model Runs*											
(0.78,10)	312.1	0.362	0.63	1.12	69.96	38.25	13.88	1.204	-0.801	3.15	277.5
(0.78,15)	312.2	0.360	0.68	1.12	70.84	36.14	14.17	1.409	-0.895	4.17	270.5
(0.78,25)	316.8	0.295	0.72	1.12	71.80	34.22	14.46	1.642	-1.001	5.51	263.7
(0.85,5)	313.9	0.336	0.56	1.13	69.48	41.21	13.60	0.991	-0.747	2.59	288.7
(0.92,5)	312.1	0.363	0.60	1.14	70.71	39.41	14.00	1.144	-0.871	3.75	285.6
(0.96,5)	315.5	0.313	0.63	1.14	71.52	38.36	14.25	1.248	-0.954	4.67	283.8
Semi-Optimized [i] $\Psi_{\rm MD}$ *Clumping Model Run*											
(0.768,14)	311.7	0.369	0.66	1.12	70.37	36.91	14.03	1.324	-0.845	3.63	272.4
Comparison Values from Best-Fit [j] *flat* ΛCDM *Model* ($\Omega_\Lambda = 0.713 = 1 - \Omega_{\rm M}$)											
$\cdots$	311.9	0.380	$\cdots$	1.0	69.96	69.96	13.64	0.287	-0.713	1.0	285.4
Comparison Values from Best-Fit [k] *flat SCDM Model* ($\Omega_\Lambda = 0$, $\Omega_{\rm M} = 1$)											
$\cdots$	608.2	3.4E-22	$\cdots$	1.0	61.35	61.35	10.62	1.0	0.0	1.0	287.3

[a] $\chi^2_{\rm Fit}$ computed versus the SCP Union SNe data (Kowalski et al. 2008). As discussed in the text, the $\Psi_{\rm MD}$ and $\Psi_{\rm Lin}$ runs used here for study are not absolutely optimized (i.e., $\chi^2_{\rm Fit}$ minimized) with respect to their model parameters, $(\Psi_0, z_{\rm init})$, as the ΛCDM model quoted here has been fully optimized with respect to Ω_Λ.

[b] Each likelihood probability $P_{\rm Fit}$ is derived from the corresponding $\chi^2_{\rm Fit}$ using the $\chi^2_{N_{\rm DoF}}$ distribution with $N_{\rm DoF}$ degrees of freedom, where $N_{\rm DoF} = 304$ for our $\Psi_{\rm Lin}$ and $\Psi_{\rm MD}$ clumping models, $N_{\rm DoF} = 305$ for the flat ΛCDM model, and $N_{\rm DoF} = 306$ for flat SCDM.

[c] The integrated (Newtonian) gravitational perturbation potential at t_0, as computed via Equation 12.

[d] Each $z^{\rm Obs}$ quoted here corresponds to $z^{\rm FRW} \equiv 1$.

[e] The $H_0^{\rm Obs}$ value (given here in km s^{-1}Mpc^{-1}) for each run is found by minimizing its $\chi^2_{\rm Fit}$ with respect to the SCP Union SNe data.

[f] Each $H_0^{\rm FRW}$ is computed relative to the corresponding optimized $H_0^{\rm Obs}$ value for that run.

[g] All $t_0^{\rm Obs}$ values are listed here in GYr, and computed assuming *no radiation* (i.e., $\Omega_R \equiv 0$).

[h] All $\Omega_{\rm M}^{\rm FRW}$ values given here for the $\Psi_{\rm Lin}$ and $\Psi_{\rm MD}$ models are normalized to $\Omega_{\rm M}^{\rm Obs} \equiv 0.27$.

[i] This "Semi-Optimized" $\Psi_{\rm MD}$ run is one chosen from a class of low-$\chi^2_{\rm Fit}$ models, as described in the text.

[j] "Best-Fit" for the flat ΛCDM Model refers here to an optimization over Ω_Λ and $H_0^{\rm Obs}$.

[k] "Best-Fit" for the flat SCDM Model refers here to an optimization over $H_0^{\rm Obs}$.